

MA 16020
Lesson 4
Integration by Parts (Part 1)

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Many of you are tempted to integrate a product by integrating its factors, but $\int u(x) \cdot v(x) dx$ is not $\int u(x) dx \cdot \int v(x) dx$.

Recall: The product rule states that $(uv)' = uv' + vu'$, or $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

Multiplying both sides by dx gives

$$d(uv) = u dv + v du$$

Integrating both sides, we get

$$\int d(uv) = \int u dv + \int v du$$

Subtracting $\int v du$ from both sides, we get

$$\int u dv = \int d(uv) - \int v du$$

$$\boxed{\int u dv = uv - \int v du}$$

Memorize this formula!

The Technique of Integration by Parts

If you are trying to integrate a product of functions, and u -substitution does not work,

1. Choose the function which is easiest to differentiate, and set it equal to u .
2. Take the remaining function and dx , and set it equal to dv .
3. Differentiate u to get du .
4. Integrate dv to get v .
5. Plug into formula $\int u dv = uv - \int v du$.
6. Finish the integral.

Oftentimes, the best function to choose for u follows

L A T E
S O G R X
t o o p o s s i b l e

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Ex 1. $\int_1^3 x(x-2)^6 dx$

It's easy to differentiate x , so let $u=x$

Then let $dv = (x-2)^6 dx$

$\frac{du}{dx} = 1$, so $du = dx$

$\int dv = \int (x-2)^6 dx = \frac{1}{7}(x-2)^7$, so $v = \frac{1}{7}(x-2)^7$

$$\int u dv = uv - \int v du$$

$$\begin{aligned} \int_1^3 x(x-2)^6 dx &= \left(x \cdot \frac{1}{7}(x-2)^7 - \int \frac{1}{7}(x-2)^7 dx \right) \Big|_1^3 \\ &= \left(\frac{x(x-2)^7}{7} - \frac{1}{56}(x-2)^8 \right) \Big|_1^3 \end{aligned}$$

$$= \left(\frac{(3)(1)^7}{7} - \frac{(1)^8}{56} \right) - \left(\frac{(1)(-1)^7}{7} - \frac{(-1)^8}{56} \right)$$

$$= \frac{3}{7} - \frac{1}{56} + \frac{1}{7} + \frac{1}{56} = \boxed{\frac{4}{7}}$$

Ex 2. $\int (t-1)e^{1-t} dt$

By LATE, $u = t-1$

so $dv = e^{1-t} dt$

$\frac{du}{dt} = 1$, so $du = dt$

$v = \int dv = \int e^{1-t} dt$ ($w = 1-t$, $dw = -dt$, $-dw = dt$)
 $-\int e^w dw = -e^w = -e^{1-t}$, so $v = -e^{1-t}$

$$\int u dv = uv - \int v du$$

$$\begin{aligned} \int (t-1)e^{1-t} dt &= (t-1)(-e^{1-t}) - \int -e^{1-t} dt \\ &= \boxed{-(t-1)e^{1-t} - e^{1-t} + C} \end{aligned}$$

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Ex 3. $\int 5x \cos(3x) dx$

By LATE, $u = 5x$

so $dv = \cos(3x) dx$

$\frac{du}{dx} = 5$, so $du = 5 dx$

$v = \int dv = \int \cos(3x) dx$ ($w = 3x$, $dw = 3 dx$, $\frac{1}{3} dw = dx$)

$\frac{1}{3} \int \cos(w) dw = \frac{1}{3} \sin(w) = \frac{1}{3} \sin(3x)$; $v = \frac{1}{3} \sin(3x)$

$\int u dv = uv - \int v du$

$\int 5x \cos(3x) = 5x \cdot \frac{1}{3} \sin(3x) - \int \frac{1}{3} \sin(3x) \cdot 5 dx$

$= \frac{5}{3} x \sin(3x) - \frac{5}{3} \int \sin(3x) dx$

$w = 3x$, $dw = 3 dx$, $\frac{1}{3} dw = dx$

$\int \sin(3x) dx = \frac{1}{3} \int \sin(w) dw = -\frac{1}{3} \cos(3x)$

$\frac{5}{3} x \sin(3x) - \frac{5}{3} \left(-\frac{1}{3} \cos(3x) \right) + C$

$\boxed{\frac{5}{3} x \sin(3x) + \frac{5}{9} \cos(3x) + C}$

Ex 4. $\int 7x \ln(x^8) dx$

By LATE, $u = \ln(x^8)$

so $dv = 7x dx$

$\frac{du}{dx} = \frac{1}{x^8} \cdot 8x^7 = \frac{8}{x}$, so $du = \frac{8}{x} dx$

$v = \int dv = \int 7x dx = \frac{7}{2} x^2$, so $v = \frac{7}{2} x^2$

$\int u dv = uv - \int v du$

$\int 7x \ln(x^8) dx = \ln(x^8) \cdot \frac{7}{2} x^2 - \int \frac{7}{2} x^2 \cdot \frac{8}{x} dx$

$= \frac{7}{2} x^2 \ln(x^8) - 28 \int x dx$

$= \frac{7}{2} x^2 \ln(x^8) - 28 \cdot \frac{1}{2} x^2 + C$

$= \boxed{\frac{7}{2} x^2 \ln(x^8) - 14x^2 + C}$

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Ex 5. $\int \frac{3x^3}{\sqrt{7+x^2}} dx$

$$\int 3x^2 \cdot x (7+x^2)^{-1/2} dx$$

$$u = 3x^2$$

$$du = 6x dx$$

$$dv = x (7+x^2)^{-1/2} dx$$

$$w = 7+x^2, dw = 2x dx, \frac{1}{2} dw = x dx$$

$$\frac{1}{2} \int w^{-1/2} dw = w^{1/2} = (7+x^2)^{1/2}$$

$$v = (7+x^2)^{1/2}$$

$$\begin{aligned} \int \frac{3x^3}{\sqrt{7+x^2}} dx &= 3x^2 \cdot (7+x^2)^{1/2} - \int (7+x^2)^{1/2} \cdot 6x dx \\ &= 3x^2 \sqrt{7+x^2} - 6 \int x (7+x^2)^{1/2} dx \end{aligned}$$

$$u = 7+x^2$$

$$du = 2x dx, \text{ so } \frac{1}{2} du = x dx$$

$$\int x (7+x^2)^{1/2} dx = \frac{1}{2} \int u^{1/2} du$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2}$$

$$= \frac{1}{3} (7+x^2)^{3/2}$$

$$3x^2 \sqrt{7+x^2} - 6 \left(\frac{1}{3} (7+x^2)^{3/2} \right) + C$$

$$\boxed{3x^2 \sqrt{7+x^2} - 2(7+x^2)^{3/2} + C}$$

Sometimes you may need to perform integration by parts twice (doing it again on $\int v du$, for example)

$$\int (6z^2 + 5) e^z dz \text{ is an example.}$$