A 600 gallon tank initially contains 400 gallons of pure distilled water. Brine containing 3 pounds of salt per gallon flows into the tank at a rate of 4 gallons per minute, and the well-stirred mixture flows out at a rate of 1 gallon per minutes. Set up a differential equation representing this scenario.

<u>Solution</u>.

We do the same thing as in example 1 in class: Let $A(t)$ represent the amount of salt (in lbs) in the tank at time t (in minutes). Then $\frac{dA}{dt}$ = (rate of amount of salt into tank) − (rate of amount of salt out of tank).

The rate of salt into the tank is the concentration of the salt flowing into the tank times the flow rate into the tank. The concentration is $\frac{3 \text{ lbs}}{\text{gal}}$, and the flow rate is $\frac{4 \text{ gal}}{\text{min}}$. Thus, the rate of amount of salt into the tank is 12 lbs/min.

Since the tank is well-stirred, the concentration of the salt flowing out of the tank is the same as the concentration of the salt in the tank as a whole. We know the units of concentration is lbs/gal. Looking at the entire tank, we know that at any time t, there are $A(t)$ lbs of salt in the tank. Now, we divide by the volume of the mixture in the tank.

<u>Notice</u>: Since the rate of flow of liquid into the tank is *different* from the rate of flow of the liquid out of the tank, the volume is *not* constant. Rather, since 4 gallons flows in per minute, and since 1 gallon flows out per minute, the rate of change of the volume is $4-1=3$ gallons per minute. Thus, we have the differential equation $\frac{dV}{dt}=3$. Solving this differential equation (using separation of variables), we get $V=3t+C$. Knowing that the tank starts with 400 gallons of mixture in it, we know that when $t=0$, $V=400$. Thus, we get $400=C$, so $V=3t+400$. (You probably have guessed this equation for the volume without thinking of the information above as a differential equation.)

And of course, the flow rate out of the tank is $\frac{1 \text{ gal}}{\text{min}}$.

Therefore, the rate of the amount of salt out of the tank is $\frac{A \text{ lbs}}{400+3t \text{ gal}} \cdot \frac{1 \text{ gal}}{\text{min}} = \frac{A}{400+3t}$ lbs/min.

Thus, taking the rate of the amount of salt into the tank minus the rate of the amount of salt out of the tank, we get

$$\frac{dA}{dt} = 12 - \frac{A}{400+3t}$$

<u>Fun fact</u>: This differential equation is *not* separable, so you cannot use separation of variables to solve it. We will see how to solve differential equations like this in lessons 9 and 10.