Eddie Price **Limit Guide** **Spring 2017**

Limits are a way to describe what the behavior of a function should be if were to continue in the way it is going. Often, you can tell what the limit is by analyzing the graph. We will cover some important limits here.

If a function is continuous, then $\lim_{x \to c} f(x) = f(c)$. So, for example $\lim_{x \to 3} x^2 = 3^2 = 9$, or $\lim_{x \to 4} e^x = e^4$.

If $\lim_{x \to c} f(x) = \pm\infty$, then $\lim_{x \to c} \frac{1}{f(x)} = 0$. For example, since $\lim_{x \to \infty} x = \infty$, we know that $\lim_{x \to \infty} \frac{1}{x} = 0$.

Here is a graph of e^x:

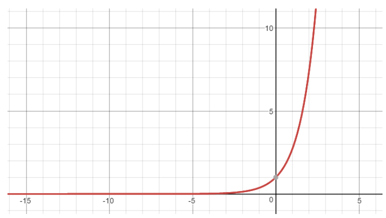

Figure 1: Made in Desmos

From here, we see the following limits: $\lim_{x \to \infty} e^x = \infty$ and $\lim_{x \to -\infty} e^x = 0$. Knowing that $e^{-x} = \frac{1}{e^x}$ we then get that $\lim_{x \to \infty} e^{-x} = 0$ and $\lim_{x \to -\infty} e^{-x} = \infty$.

Here is a graph of $\ln(x)$:

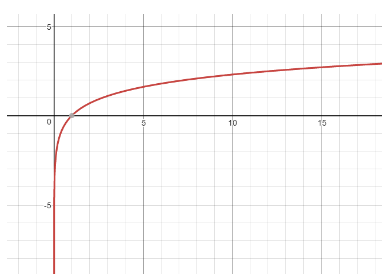

Figure 2: Made in Desmos

It may be difficult to see from the graph, but we get: $\lim_{x \to \infty} \ln(x) = \infty$ and $\lim_{x \to 0^+} \ln(x) = -\infty$.

Important fact: it is well-known (which requires more powerful tools than we have access to in this class) that exponential functions "beat" polynomials in terms of limits. So for example, $\lim_{x \to \infty} xe^{-x} = 0$ since exponential functions beat polynomials and $\lim_{x \to \infty} e^{-x} = 0$.

WARNING: $0 \cdot \infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$ and $\infty - \infty$ are *undefined*. If you get an expression like this while taking a limit, you have to use another limit fact (for example, $\lim_{x \to \infty} xe^{-x} = 0$ since exponential functions beat polynomials, even though naively this limit looks like $\infty \cdot 0$).