Eddie Price **Quiz 13 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 1; Average score: 7.66 (including 0's)

Problem 1 (10 Points). Find all critical points of $f(x, y)$ and classify them as relative minima, relative maxima, or saddle points.

$$f(x, y) = 4 + x^3 + y^3 - 3xy$$

Solution. We find $f_x = 3x^2 - 3y$ and $f_y = 3y^2 - 3x$.

To find the critical points, we must set both of these partial derivatives equal to 0: $3x^2 - 3y = 0$ and $3y^2 - 3x = 0$. We can solve for y in the first equation and get $y = x^2$. We may now substitute this into the second equation, to get $3\left(x^2\right)^2 - 3x = 0$, i.e., $3x^4 - 3x = 0$. We can factor out $3x$ to get $3x\left(x^3 - 1\right) = 0$. So we get $x = 0$ or $x^3 - 1 = 0$ (equivalent to $x^3 = 1$ equivalent to $x = 1$). So $x = 0$ or $x = 1$.

Since $y = x^2$, we get that $y = (0)^2 = 0$ when $x = 0$, so we get the critical point $(0, 0)$. Also, $y = (1)^2 = 1$ when $x = 1$, so we get the critical point $(1, 1)$.

We now find $D(x, y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$. Now, $f_{xx} = 6x$, $f_{yy} = 6y$, and $f_{xy} = -3$, so $D(x, y) = 36xy - 9$.

$D(0, 0) = -9 < 0$, so f has a $\boxed{\text{saddle point at } (0, 0)}$.

$D(1, 1) = 36 - 9 = 27 > 0$, so we check $f_{xx}(1, 1) = 6 > 0$, so f has a $\boxed{\text{relative minimum at } (1, 1)}$.

Common Mistakes

Many people had trouble finding the critical points or they got extra critical points.

One mistake made when trying to find the critical points was dividing both sides of an equation by a variable and canceling that variable. Cancellation like this can cause you to lose solutions. Do not do it.