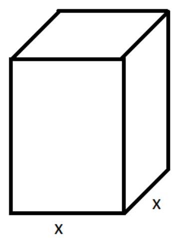
Eddie Price **Quiz 14 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 1; Average score: 7.67 (including 0's)

Problem 1 (10 Points). A rectangular box with a square base is to be constructed from materials that cost \$3/ft^2 for the top, \$2/ft^2 for the base, and \$5/ft^2 for the sides. What is the minimum cost to construct such a box which has a volume of 8 ft^2? (Round to the nearest cent.)

Solution. The box has a square base, and so looks something like this:



The cost is given by $3x^2$ for the top (since it has an area of x^2 and costs \$3 per square foot), $2x^2$ for the base (since it has an area of x^2 and costs \$2 per square foot), and $20xy$ for the sides (since there are 4 sides, each with an area of xy, and each costs \$5 per square foot). So $C = 5x^2 + 20xy$. This is what we are optimizing. The volume is x^2y and is equal to 8, so our constraint is $x^2y = 8$.

$$C_x = 10x + 20y, \quad C_y = 20x, \quad V_x = 2xy, \quad V_y = x^2$$

So we get the system of equations

$$10x + 20y = \lambda 2xy, \quad 20x = \lambda x^2, \quad x^2y = 8$$

Solving the second equation, we get $x = 0$ (which is impossible, due to the third equation) or $\lambda = \frac{20}{x}$, which we can plug into the first equation to get $10x + 20y = 40y$, or equivalently, $x = 2y$. We can now plug this into the constraint to get $(2y)^2\, y = 8$, or $4y^3 = 8$, giving that $y = \sqrt[3]{2}$. Plugging back into $x = 2y$, we see $x = 2\sqrt[3]{2}$.

We plug into the cost function $C = 5x^2 + 20xy = 5\left(2\sqrt[3]{2}\right)^2 + 20 \cdot 2\sqrt[3]{2} \cdot \sqrt[3]{2} \approx \boxed{\$95.24}$.

Common Mistakes

Many people messed up the cost function.

Many people made algebra errors.