Eddie Price **Quiz 17 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 3; Average score: 7.39 (including 0's)

Problem 1 (10 Points). Solve the following system of equations using Gaussian Elimination:

$$\begin{cases} x - y + 2z = -1 \\ 3x + y - z = 4 \\ -x + 2y - z = 0 \end{cases}$$

Clearly show each row operation that you perform. (You may also use Gauss-Jordan Elimination if you wish)

Solution. The augmented matrix for the system is

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 3 & 1 & -1 & 4 \\ -1 & 2 & -1 & 0 \end{array}\right]$$

Next, we use elementary row operations to transform the matrix into row-echelon form:

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 3 & 1 & -1 & 4 \\ -1 & 2 & -1 & 0 \end{array}\right] \xrightarrow{-3R_1+R_2\mapsto R_2} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 4 & -7 & 7 \\ -1 & 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_1+R_3\mapsto R_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 4 & -7 & 7 \\ 0 & 1 & 1 & -1 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 4 & -7 & 7 \end{array}\right]$$

$$\xrightarrow{-4R_2+R_3\mapsto R_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -11 & 11 \end{array}\right] \xrightarrow{-\frac{1}{11}R_3\mapsto R_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

So we have the new system of equations

$$\begin{cases} x - y + 2z = -1 \\ \quad\;\; y + \; z = -1 \\ \qquad\quad\; z = -1 \end{cases}$$

We plug $z = -1$ into $y + z = -1$ to get $y + (-1) = -1$, giving $y = 0$. We now plug in $z = -1$ and $y = 0$ to the equation $x - y + 2z = -1$ to get $x - 0 - 2 = -1$, i.e., $x - 2 = -1$, which gives $x = 1$.

So the solution is $\boxed{x = 1,\ y = 0,\ z = -1}$ or $\boxed{(1, 0, -1)}$.

Common Mistakes

People made algebra errors.

People also performed row operations which undid some of their earlier work to get 0's in certain places.