

High score: 10; Non-0 Low score: 1; Average score: 7.79 (including 0's)

Problem 1 (8 Points). Compute $\det(A)$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & 3 \\ 0 & 4 & -3 \end{bmatrix}$$

Solution. You can do a cofactor expansion along any row or any column. It seems easiest to cofactor expand along column 1. As such, $\det(A) = 1 \cdot C_{11} + 2 \cdot C_{21} + 0 \cdot C_{31} = C_{11} + 2C_{21}$. Thus, we need to find C_{11} and C_{21} . (Remember: the first number always refers to the *row* number, and the second number always refers to the *column* number.)

$$M_{11} = \begin{vmatrix} \cancel{1} & \cancel{2} & \cancel{-1} \\ 2 & -2 & 3 \\ 0 & 4 & -3 \end{vmatrix} = \begin{vmatrix} -2 & 3 \\ 4 & -3 \end{vmatrix} = (-2)(-3) - (4)(3) = 6 - 12 = -6.$$

$$C_{11} = (-1)^{1+1} M_{11} = (-1)^2 (-6) = -6.$$

$$M_{21} = \begin{vmatrix} \cancel{1} & \cancel{2} & \cancel{-1} \\ \cancel{2} & \cancel{-2} & \cancel{3} \\ 0 & 4 & -3 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 4 & -3 \end{vmatrix} = (2)(-3) - (4)(-1) = -6 - (-4) = -6 + 4 = -2$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-2) = 2$$

$$\text{So } \det(A) = C_{11} + 2C_{21} = (-6) + 2(2) = -6 + 4 = \boxed{-2}.$$

Problem 2 (2 Points). Is B nonsingular (i.e., invertible) or singular (i.e., not invertible)?

$$B = \begin{bmatrix} 1 & -4 \\ -2 & 8 \end{bmatrix}$$

Solution.

$\det(B) = (1)(8) - (-2)(-4) = 8 - 8 = 0$. Thus, A is singular (i.e., not invertible).

Common Mistakes

Some people multiplied 0 times something and got something other than 0.