Eddie Price **Quiz 1 Solutions** **Spring 2017**

High score: 10; Low score: 1; Average score: 8.31

Problem 1 (5 Points). Evaluate the definite integral. Give the *exact* answer.

$$\int_0^{\pi/2} (\sin x + x)\, dx$$

Solution. First, notice that you can integrate terms separately, so we obtain

$$\left(-\cos x + \frac{1}{2}x^2\right)\Big|_0^{\pi/2}$$

$$\left(-\cos\left(\frac{\pi}{2}\right) + \frac{1}{2}\left(\frac{\pi}{2}\right)^2\right) - \left(-\cos(0) + \frac{1}{2}(0)^2\right)$$

$$\left(0 + \frac{1}{2}\frac{\pi^2}{4}\right) - (-1 + 0)$$

$$\boxed{\frac{\pi^2}{8} + 1}$$

Problem 2 (5 points). Evaluate the indefinite integral.

$$\int \frac{1}{\sqrt{x}} \cos\left(3 + \sqrt{x}\right) dx$$

Solution. Notice that this looks like the result of the chain rule, so we use substitution. Notice that the innermost function is $3 + \sqrt{x}$, so we set

$$u = 3 + \sqrt{x} = 3 + x^{1/2}$$

Hence, $\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$. Thus, $2du = \frac{1}{\sqrt{x}}dx$.
We can rewrite our integral as

$$\int \cos\left(3 + \sqrt{x}\right) \cdot \frac{1}{\sqrt{x}}dx$$

Substituting $u = 3 + \sqrt{x}$ and $2du = \frac{1}{\sqrt{x}}dx$, we obtain the new integral

$$2\int \cos(u)\, du$$

$$2\sin(u) + C$$

Since $u = 3 + \sqrt{x}$, we have

$$\boxed{2\sin\left(3 + \sqrt{x}\right) + C}$$

<u>Common Mistakes</u>

For both problems, people confused the signs for integrating $\sin x$ and $\cos x$. Remember that the derivative of $\sin x$ is $\cos x$; thus, the integral of $\cos x$ is $\sin x + C$. Similarly, the derivative of $\cos x$ is $-\sin x$; thus, the integral of $\sin x = -\cos x + C$.

For both problems, people confused integration with differentiation. For problem 1, some people thought that the integral of x was 1 (they were accidentally taking the derivative), and for problem 2, some people thought that given $u = 3 + x^{1/2}$, the derivative $\frac{du}{dx} = 3x + \frac{2}{3}x^{3/2}$ or some variant (they were accidentally integrating).

For problem 1, many people forgot their unit circle values.

For problem 1, many people tried to use substitution when it was unnecessary and did not work.

For problem 1, many people included a "$+C$" in their answer. This is incorrect since a definite integral has a definite value. The $+C$ comes from antidifferentiating, and when actually evaluating an integral, will cancel itself out. For instance, if $F(x)$ is an antiderivative of $f(x)$, then so is $F(x) + C$, but $(F(b) + C) - (F(a) + C) = F(b) - F(a)$.

For problem 2, many people forgot the "$+C$" in their answer. This is an indefinite integral, meaning we must include $+C$. (See lesson R.)

For both problems, many people made algebra errors.