High score: 10; Low score: 0 (non-0 score: 4); Average score: 7.52 (including 0's)

Problem 1 (5 Points). Evaluate the definite integral. Give the *exact* answer.

$$\int_0^1 \frac{2}{(2x+1)^2} dx$$

Solution. An inner function here is $2x+1$ and its derivative is a constant, so we can use this as our substitution. Let $u = 2x+1$. Then $\frac{du}{dx} = 2$, so $du = 2dx$. In other words, $\frac{1}{2}du = dx$.

Also, notice that for the lower bound of the integral, we plug $x = 0$ into the equation $u = 2x+1$, giving $2(0)+1 = 1$.
For the upper bound of the integral, we plug $x = 1$ into the equation $u = 2x+1$, giving $2(1)+1 = 3$.
Now, we make the substitution:

$$\int_a^b \frac{2}{u^2} \cdot \frac{1}{2}\, du$$

$$\int_1^3 u^{-2}\, du$$

$$-u^{-1}\ \Big|_1^3$$

$$-(3)^{-1} - \left(-(1)^{-1}\right)$$

$$-\frac{1}{3} + 1$$

$$\boxed{\frac{2}{3}}$$

(Note: You can also get find that the antiderivative, in terms of u, is $-u^{-1}$, convert back into terms of x to get $-(2x+1)^{-1}$ and plugged in the original upper bound 1 and original lower bound 0 to obtain the answer.)

Problem 2 (5 points). Evaluate the indefinite integral.

$$\int x\sqrt{x-1}\,dx$$

Solution. Notice that $x-1$ is inside of another function, so try $u = x-1$. Then $\frac{du}{dx} = 1$, so $du = dx$. Making our substitution, we obtain

$$\int x\sqrt{u}\,du$$

Here, we have a left-over x in the integral, which me must convert to be a function in terms of u. For this, we use the original substitution equation $u = x-1$ and add 1 to both sides to get $u+1 = x$. Now, we can substitute $u+1$ for x in the integral to get

$$\int (u+1)\sqrt{u}\,du$$

$$\int (u+1)\,u^{1/2}\,du$$

$$\int \left(u^{3/2} + u^{1/2}\right)\,du$$

$$\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$$

We must convert our function back so that the variable is once again x, so now we make the substitution $u = x-1$ to get

$$\boxed{\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C}$$

Common Mistakes

For problem 1, some people converted the bounds of the integral (0 and 1) into the u-values 1 and 3, but also converted their antiderivative back into a function of x and plugged the u-value bounds in for x. When you have an integral of the form $\int_a^b f(x)\;dx$, a and b are x-values and can only be plugged into x. If you make a u-substitution and convert a and b using the "$u =$" equation, then the bounds are u-values and can only be plugged into u. You cannot plug x-values into u or u-values into x.

For problem 1, several people integrated u^{-2} incorrectly. Recall the power rule for integrals: $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ (for $n \neq -1$), so $\int u^{-2}\,du = \frac{1}{-2+1}u^{-2+1} + C = -u^{-1} + C$.

For problem 2, many people correctly identified that $u = x-1$, and noticed a left over x in the integral after making the substitution. To solve for x, you *add* 1 to both sides, giving $u+1 = x$. Many people put $u-1 = x$, which is an algebra error.

For problem 2, many people tried to integrate a product of functions by integrating the factors separately. Similar to the product rule where $(u \cdot v)' \neq u' \cdot v'$, we also have the same for integrals $\int u \cdot v \, dx \neq \int u \, dx \cdot \int v \, dx$. You *cannot* integrate factors separately.

For problem 2, many people forgot to substitute $u = x - 1$ back into the function after integrating. For an indefinite integral, if you start with a function whose variable is x, you have to end with a function whose variable is x.

For problem 2, many people forgot their $+C$. See Lesson R for why this is important.