Eddie Price **Quiz 3 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 1; Average score: 7.8 (including 0's)

<u>Problem 1</u> (5 Points). Evaluate the indefinite integral.

$$\int x\sin(x)\ dx$$

<u>Solution</u>. We do not know an antiderivative for this automatically, and substitution will not help, so we use integration by parts. A choice of $u = x$ and $dv = \sin x\, dx$ will allow all the integrals we have to compute to actually be doable.

Since $u = x$, we get that $du = dx$, and since $dv = \sin x\, dx$, we have that $v = \int dv = \int \sin x\, dx = -\cos x$. Using the formula $\int u\, dv = uv - \int v\, du$, we get

$$\int x\sin x\, dx = (x)(-\cos x) - \int -\cos x\, dx$$

Now, $\int -\cos x\, dx = -\sin x$, so we get

$$-x\cos x - (-\sin x)$$

We simplify and notice that this is an *indefinite* integral, so we also add C to get

$$\boxed{-x\cos x + \sin x + C}$$

Problem 2 (5 points). Evaluate the indefinite integral.

$$\int x^2 \ln(3x)\ dx$$

Solution. We don't know an antiderivative and substitution will not make the integral any simpler, so we use integration by parts. We do not know how to integrate $\ln(3x)$ without using integration by parts, so we should choose $u = \ln(3x)$ and $dv = x^2\,dx$.

Thus, $du = \frac{1}{3x} \cdot 3\,dx = \frac{1}{x}\,dx$ and $v = \int dv = \int x^2\,dx = \frac{1}{3}x^3$. Plugging into the formula $\int u\,dv = uv - \int v\,du$, we get

$$\begin{aligned} \int x^2 \ln(3x)\ dx &= \ln(3x)\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx \\ &= \frac{1}{3}x^3 \ln(3x) - \frac{1}{3}\int x^2\,dx \end{aligned}$$

Now, $\int x^2\,dx = \frac{1}{3}x^3$, and noticing that we have an indefinite integral, we get

$$\frac{1}{3}x^3 \ln(3x) - \frac{1}{3}\left(\frac{1}{3}x^3\right) + C$$

$$\boxed{\frac{1}{3}x^3 \ln(3x) - \frac{1}{9}x^3 + C}$$

Common Mistakes

For problem 1, many people made mistakes on the sign for integrating $\sin x$ or $\cos x$.

For problem 2, many people thought that $\frac{1}{x}$ was the *antiderivative* of $\ln(3x)$, when it is really the derivative. This caused them to select the incorrect u function. For this problem, select $u = \ln(3x)$.

For problem 2, when taking the derivative of $\ln(3x)$, use the chain rule: $\frac{1}{3x} \cdot 3 = \frac{1}{x}$. Many people forgot to use the chain rule here. An alternate way to see this is to use log rules: $\ln(3x) = \ln(3) + \ln(x)$, which has derivative $0 + \frac{1}{x}$ (since $\ln(3)$ is a constant).