Eddie Price **Quiz 4 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 2; Average score: 7.88 (including 0's)

Problem 1 (10 Points). Newton's Law of Cooling states that an object's temperature changes at a rate proportional to the difference between its temperature and the temperature of the surrounding area. A pastry chef is preparing a pastry for a patron at her restaurant. When she removes the pastry from the oven, its temperature is 200°F. She tests the temperature of the pastry 1 minute later and finds that it is 190°F. If the temperature of the kitchen is 75°F, and if the pastry is best served when its temperature is 150°F, how many more minutes should the pastry chef wait before serving the pastry? Round to 2 decimal places.

Solution. Let T be temperature of the pastry with respect to t being time measured in minutes. The surrounding temperature is 75°F, so the difference between the pastry's temperature and the surrounding temperature is $T - 75$. Since the temperature changes at a rate proportional to $T - 75$, we get the differential equation

$$\frac{dT}{dt} = k(T - 75)$$

Using separation of variables, we get

$$\frac{1}{T - 75}\, dT = k\, dt$$

Integrating both sides, we get

$$\ln|T - 75| = kt + C$$

Exponentiating both sides, using exponentiation rules, and observing that e raised to a constant is still a constant, we get

$$|T - 75| = e^{kt+C} = e^C e^{kt} = Ce^{kt}$$

Since plus or minus a constant is still a constant, we can get rid of the absolute values and add 75 to both sides to obtain

$$T = Ce^{kt} + 75$$

When the pastry is removed from the oven, its temperature is 200°F, so when $t = 0$, $T = 200$:

$$200 = Ce^0 + 75 = C + 75$$

Giving that $C = 200 - 75 = 125$. Thus, we now have

$$T = 125e^{kt} + 75$$

Now, we must find k. We know that 1 minute after being pulled out of the oven, the pastry is 190°F, so when $t = 1$, $T = 190$, giving:

$$190 = 125e^{k} + 75$$

Subtracting 75 from both sides, dividing both sides by 125, and then taking the natural log of both sides, we see

$$k = \ln\left(\frac{115}{125}\right)$$

Thus, we get the equation

$$T = 125e^{\ln\left(\frac{115}{125}\right)t} + 75$$

We want to know when the temperature is 150°F, so we set $T = 150$ and solve for t:

$$150 = 125e^{\ln\left(\frac{115}{125}\right)t} + 75$$

Subtracting 75 from both sides, dividing both sides by 125, and then taking the natural log of both sides, we see

$$\ln\left(\frac{75}{125}\right) = \ln\left(\frac{115}{125}\right)t$$

Thus, we get

$$t = \frac{\ln\left(\frac{75}{125}\right)}{\ln\left(\frac{115}{125}\right)} \approx 6.13$$

Thus, it takes approximately 6.13 minutes for the pastry to reach the proper temperature after it is removed from the oven. But we want to know how many <u>more</u> minutes she has to wait after having checked the time at 1 minutes after being removed from the oven. Thus, she has to wait $6.13 - 1 = 5.13$ more minutes.

$$\boxed{\text{5.13 more minutes}}$$

<u>Common Mistakes</u>

Many people used the wrong formula for Newton's Law of Cooling.

There were algebra errors.

There were calculator input errors.