Eddie Price **Quiz 5 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 2; Average score: 8.27 (including 0's)

<u>Problem 1</u> (10 Points). A wet towel hung on a clothesline to dry outside loses moisture at a rate proportional to its moisture content. After 1 hour, the towel has lost 41% of its original moisture content. After how long will the towel have lost 72% of its moisture content? Round your answer to two decimal places.

<u>Solution</u>. Let $M(t)$ represent the moisture content of the towel after t hours. Since the rate of change of M is proportional to M, we have the differential equation

$$\frac{dM}{dt} = kM$$

Separating variables, we get

$$\frac{1}{M} \, dM = k \, dt$$

Integrating and simplifying, we get the equation

$$M(t) = Ce^{kt}$$

At $t = 0$, the towel has 100% of its original moisture content; so when $t = 0$, $M = 100\% = 1$.

$$1 = Ce^0 = C$$

So we have $M(t) = e^{kt}$.

Now, when $t = 1$, the towel has <u>lost</u> 41% of its original moisture content, so it has $100\% - 41\% = 59\%$ of its original moisture content. Thus, when $t = 1$, $M = 59\% = 0.59$

$$0.59 = e^k$$

Taking the natural log of both sides, we see $k = \ln(0.59)$. Hence, we have

$$M(t) = e^{\ln(0.59)\cdot t}$$

When the towel has lost 72% of its moisture content, it has $100\% - 72\% = 28\%$ of its moisture content. So we want to find t when $M = 28\% = 0.28$.

$$0.28 = e^{\ln(0.59)\cdot t}$$

Taking the natural log of both sides, we get

$$\ln(0.28) = \ln(0.59) \cdot t$$

$$t = \frac{\ln(0.28)}{\ln(0.59)} \approx 2.41$$

$$\boxed{2.41 \text{ hours}}$$

Common Mistakes

The most common mistake was using 0.41 instead of 0.59 and using 0.72 instead of 0.28.

Some people did not convert their percentages into decimals.