Eddie Price **Quiz 6 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 1; Average score: 7.38 (including 0's)

Problem 1 (5 Points). Find the area of the region bounded by the curves: $x = y^2$, $y = x$

Solution. First sketch a graph of the region:

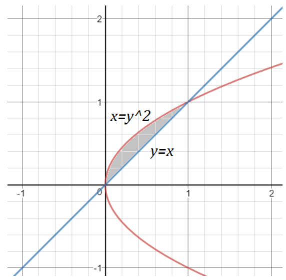

Figure 1: Plot made in Desmos

We have a right curve and a left curve here; we need everything should be in terms of y. The right curve is $x = y$ and the left curve is $x = y^2$. The points of intersection are $y = 0$ and $y = 1$: $y^2 = y$ is true when $y^2 - y = 0$, which is true when $y\,(y-1) = 0$, giving $y = 0$ and $y = 1$. Thus, the area between the curves can be found by taking the integral:

$$\begin{aligned} A &= \int_0^1 \left(y - y^2\right)\, dy \\ &= \frac{1}{2}y^2 - \frac{1}{3}y^3 \,\Big|_0^1 \\ &= \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0) \\ &= \frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}} \end{aligned}$$

OR We also have a top curve and a bottom curve; we need everything in terms of x: $y = \pm\sqrt{x}$ and $y = x$. Since we're only dealing with positive y-values here, we have the function $y = \sqrt{x}$. The top curve is $y = \sqrt{x}$ and the bottom curve is $y = x$. The points of intersection again are $x = 0$ and $x = 1$: $\sqrt{x} = x$ is true when $x = x^2$, or $x^2 - x = 0$, which is when $x = 0$ or $x = 1$. Thus, the area is given by the integral:

$$\begin{aligned} A &= \int_0^1 \left(\sqrt{x} - x\right)\, dx \\ &= \frac{2}{3}x^{3/2} - \frac{1}{2}x^2 \,\Big|_0^1 \\ &= \left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0) \\ &= \frac{4}{6} - \frac{3}{6} = \boxed{\frac{1}{6}} \end{aligned}$$

Problem 2 (5 Points). Find the area of the region bounded by the curves: $y = e^x$, $y = e^{-x}$, $x = 0$, $x = \ln(2)$

Solution. First sketch a graph of the region:

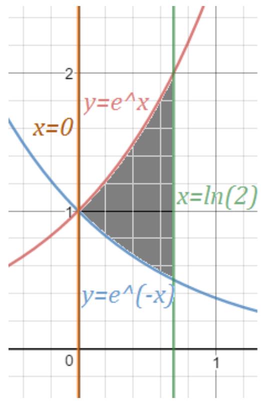

Figure 2: Plot made in Desmos

We have an upper curve and a lower curve here; we need everything should be in terms of x. The upper curve is $y = e^x$ and the lower curve is $y = e^{-x}$. From the sketch, the bounds of integration are $x = 0$ and $x = \ln(2)$. Thus, the area between the curves can be found by taking the integral:

$$\begin{aligned} A &= \int_0^{\ln(2)} \left(e^x - e^{-x}\right)\, dx \\ &= e^x + e^{-x} \Big|_0^{\ln(2)} \\ &= \left(e^{\ln(2)} + e^{-\ln(2)}\right) - \left(e^0 + e^{-0}\right) \\ &= \left(2 + e^{\ln(1/2)}\right) - (1+1) \\ &= \left(2 + \frac{1}{2}\right) - 2 \\ &= \boxed{\frac{1}{2}} \end{aligned}$$

Common Mistakes

For number 1, many people did the bottom curve minus the top curve or the left curve minus the right curve.

Many people sketched their graphs incorrectly.

For number 2, many people messed up $\int e^{-x}\, dx = -e^{-x}$

For number 2, many people thought $e^{-\ln(2)} = -2$, which is not correct.