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Quiz 8 Solutions

Spring 2017

High score: 10; Non-0 Low score: 3; Average score: 7.19 (including 0's)

<u>Problem 2</u> (10 Points). Set up (but do NOT solve) an integral representing the volume of the solid obtained by revolving the region bounded by the curves $y = \sqrt{x}$, y = x about the *y*-axis. Reminder: Do <u>not</u> evaluate the integral.

Solution. First sketch a graph of the region and the solid:



Since we're rotating about the y-axis, all functions must be in terms of y: $x = y^2$ for the parabola and x = y for the line.

From the picture, one can see that the outer radius goes from the y-axis to the line x = y. Thus, R = y. Also, the inner radius goes from the y-axis to the curve $x = y^2$, so $r = y^2$.

The washers vary from the lower intersection point of the parabola and the line until the upper intersection point (these are the lowest and highest that the washers can go, respectively). It is not difficult to see these are y = 0 to y = 1, but you can check by setting them equal: $y^2 = y \Rightarrow y^2 - y = 0 \Rightarrow y (y - 1) = 0$.

$$V = \pi \int_0^1 \left((y)^2 - (y^2)^2 \right) \, dy = \pi \int_0^1 \left(y^2 - y^4 \right) \, dy$$

Common Mistakes

Many people left their functions in terms of x. Since you're revolving about the y-axis and since the washers move up and down, you need everything in terms of y.

Many people forgot to square their radii or took $(R-r)^2$. Remember that the inside of the integral is the area of the washer, which is the area of the big circle minus the area of the little circle, $\pi R^2 - \pi r^2 = \pi (R^2 - r^2)$.

Many people switched their radii, calling y^2 the outer radius and y the inner radius. This is easy to avoid by drawing in a sample washer.

Many people forgot to multiply their integral by π . (This is necessary because of π being in the formula for the area of a circle.)

Many people got the bounds incorrect. Since the washers move up and down, look for the lowest and highest y-values in your region. (These are numbers here, not functions.)

Several people took steps toward solving the integral, even though there were two, explicit warnings not to do that.