Eddie Price **Quiz 9 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 1; Average score: 7.95 (including 0's)

Problem a (5 Points). Write the repeating decimal $0.\overline{999}$ as a geometric series in Σ notation.

Solution. First, notice that decimal notation means

$$0.\overline{999} = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + ... = \frac{9}{10^1} + \frac{9}{10^2} + \frac{9}{10^3} + ...$$

Therefore, we get

$$0.\overline{999} = \boxed{\sum_{n=1}^{\infty} \frac{9}{10^n}}$$

There are multiple other ways to write this in Σ notation, including but not limited to $\sum\limits_{n=0}^{\infty} \frac{9}{10^{n+1}}$ and $9\sum\limits_{n=1}^{\infty} \frac{1}{10^n}$, etc.

Problem b (5 Points). Find the sum of the geometric series you found in part (a).

Solution. Notice that the first term is $\frac{9}{10}$, so $a = \frac{9}{10}$. The second term is $\frac{9}{10^2}$, so $r = \frac{9}{10^2} \div \frac{9}{10} = \frac{9}{10^2} \cdot \frac{10}{9} = \frac{1}{10}$. Thus, the sum of the geometric series is

$$\frac{a}{1-r} = \frac{\frac{9}{10}}{1-\frac{1}{10}} = \frac{\frac{9}{10}}{\frac{9}{10}} = \frac{9}{10} \cdot \frac{10}{9} = \boxed{1}$$

Fun fact. This shows that $0.\overline{999} = 1$, which is a counterintuitive, but true fact.

Common Mistakes

People didn't know how to set up the series.