Eddie Price **Fast Method for 3×3 Determinants** **Spring 2017**

The method of cofactor expansion is the proper way to find a determinant for square matrices that are larger than 2×2. For 3×3 matrices, specifically, we can use the following technique. The follow technique does *not* work for square matrices of other sizes.

Take the matrix you're starting with, copy the first two *columns* of your matrix, and place them to the right of your starting matrix. This new array with 5 columns will have 3 diagonals of length 3 (a diagonal moves down and to the right) and 3 anti-diagonals of length 3 (an anti-diagonal moves up and to the right). For each diagonal and anti-diagonal, multiply together all three of the entries on it. Then add all of the products obtained from the diagonals and subtract all of the products obtained from the anti-diagonals. The number you get is the determinant.

As an example, consider the matrix A below:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

Now, copy the first two columns and place them to the right of the matrix:

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ -1 & -1 & -2 \end{bmatrix} \begin{matrix} 1 & -1 \\ 2 & 1 \\ -1 & -1 \end{matrix}$$

Look at the three diagonals of length 3:

$$\begin{matrix} 1 & -1 & 2 & 1 & -1 \\ 2 & 1 & 4 & 2 & 1 \\ -1 & -1 & -2 & -1 & -1 \end{matrix}$$

Multiply along the diagonals to get $(1)(1)(-2) = -2$, $(-1)(4)(-1) = 4$, and $(2)(2)(-1) = -4$.

Now, look at the three anti-diagonals of length 3:

$$\begin{matrix} 1 & -1 & 2 & 1 & -1 \\ 2 & 1 & 4 & 2 & 1 \\ -1 & -1 & -2 & -1 & -1 \end{matrix}$$

Multiply along the anti-diagonals to get $(-1)(1)(2) = -2$, $(-1)(4)(1) = -4$, and $(-2)(2)(-1) = 4$.

Now, add all of the products from diagonals and subtract all of the products of anti-diagonals to get

$$\det A = (-2) + (4) + (-4) - (-2) - (-4) - (4) = 0$$