MA 16020
Lesson 1

Substitution (Part 1)

Recall The chain rule. The derivative of $f(u(x))$ is $f'(u(x)) \cdot u'(x)$.

Remembering this... can tell that

$$\int e^{x^2} 2x\, dx$$

$f'(x) = e^x$, $u(x) = x^2$, $u'(x) = 2x$, $f(x) = e^x$

So $f'(u(x)) \cdot u'(x) = e^{x^2} 2x$, so we get

$$\int e^{x^2} 2x\, dx = e^{x^2} + C$$

The Substitution Technique

If it looks like the function you're taking an integral of is the result of the chain rule,

1. Let u = inside function
2. Find $\frac{du}{dx}$ and solve for du
3. Substitute u and du into the integral. Make sure you replace all x's with u's.
4. Integrate
5. Convert u's back into x's.

Ex 1. Find $\int 4e^{\cos 3x} \sin 3x\, dx$

Let $u = \cos 3x$, $\frac{du}{dx} = -\sin 3x$, so $du = -3\sin 3x\, dx$

or $-\frac{1}{3} du = \sin 3x\, dx$

$$-\frac{4}{3}\int e^u\, du = -\frac{4}{3} e^u + C$$

$$-\frac{4}{3} e^{\cos 3x} + C$$

In LON-CAPA, make sure "3x" in parentheses, C capital

Ex 2. $\int 5x^2(3x^3+2)^5\,dx$

let $u = 3x^3+2$, $\frac{du}{dx} = 9x^2$, so $du = 9x^2\,dx$

$\frac{1}{9}du = x^2dx$

$\frac{5}{9}\int u^5\,du = \frac{5}{9}\cdot\frac{1}{6}u^6 + C = \boxed{\frac{5}{54}(3x^3+2)^6 + C}$

Ex 3. A turkey is removed from the freezer. The temperature was $-2°C$ when it was removed and t hours later its temperature is increasing at a rate $T'(t) = 3e^{-0.2t}$ °C/hour. Assume it is defrosted at 11°C. How many hours (rounded to 4 decimal places) does the turkey take to defrost?

Need to find Temperature $T = \int T'(t)\,dt$

$T(t) = \int 3e^{-0.2t}\,dt$

$u = -0.2t$, $\frac{du}{dt} = -0.2$, $du = -0.2\,dt$

$-5\,du = dt$

$-15\int e^u\,du = -15e^{-0.2t} + C$

$T(t) = C - 15e^{-0.2t}$

when $t = 0$ hours, T is $-2°C$

$-2 = C - 15e^{-0.2(0)} \Rightarrow -2 = C - 15$, $C = 13$

$T(t) = 13 - 15e^{-0.2t}$

$11 = 13 - 15e^{-0.2t}$

$-2 = -15e^{-0.2t}$

$\frac{2}{15} = e^{-0.2t}$

$\ln\left(\frac{2}{15}\right) = -0.2t$

$t = -5\ln\left(\frac{2}{15}\right) \approx \boxed{10.0745 \text{ hours}}$

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Ex 4. Find a function $f(x)$ whose tangent line has the slope $\frac{1}{\sqrt{x}}\cos(3+\sqrt{x})$ and passes through the point $(1,1)$.

Slope is $f'(x)$, so $f(x) = \int \frac{1}{\sqrt{x}}\cos(3+\sqrt{x})\,dx$

$u = 3+\sqrt{x} = 3+x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$, so $2\,du = \frac{1}{\sqrt{x}}\,dx$

$2\int \cos u\,du = 2\sin u + C = 2\sin(3+\sqrt{x}) + C$

$1 = 2\sin(3+\sqrt{(1)}) + C$, so $C = 1 - 2\sin(4)$

$$\boxed{f(x) = 2\sin(3+\sqrt{x}) + 1 - 2\sin(4)}$$

Ex 5. The velocity of a bug that is moving along the t-axis is given by $v(t) = -\frac{4t}{(3+t^2)^{\frac{5}{4}}}$.

The position $s(t)$ of the bug at time $t=0$ is 8. What is $s(t)$ at time t?

$s(t) = \int -\frac{4t}{(3+t^2)^{5/4}}\,dt = -4\int \frac{t}{(3+t^2)^{5/4}}\,dt$

$u = 3+t^2$, $du = 2t\,dt \Rightarrow \frac{1}{2}du = t\,dt$

$-2\int \frac{1}{u^{5/4}}\,du = -2\int u^{-5/4}\,du$

$= -2(-4)u^{-1/4} + C = 8(3+t^2)^{-1/4} + C$

$8 = 8(3)^{-1/4} + C$, so $C = 8 - 8(3)^{-1/4}$

$$\boxed{s(t) = 8(3+t^2)^{-1/4} + 8 - 8(3)^{-1/4}}$$