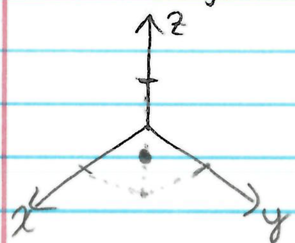
MA 16020
Lesson 18
Functions of Several Variables

$y=f(x)$ represents a 2-dimensional graph
$z=f(x,y)$ represents a 3-dimensional graph



z-values are heights above the xy-plane

Functions $z=f(x,y)$ represent surfaces instead of curves.

Given a function $z=f(x,y)$, you can find the height above the xy-plane at (a,b) by plugging in $x=a$, $y=b$.

Ex 1. Find $f(2,-1)$ for $f(x,y)=\dfrac{\ln(x-1)+y}{e^{x+y}}$

$$f(2,-1)=\frac{\ln((2)-1)+(-1)}{e^{(2)+(-1)}}=\frac{\ln(1)-1}{e^1}=\boxed{-\frac{1}{e}}$$

Domains work the same way with multiple variables: denominators can't be 0, insides of even roots must be ≥ 0, insides of logs must be >0.

Ex 2. Find the domain of $f(x,y)=\dfrac{\sqrt{x-18}}{\ln(y-5)-6}$

Need $x-18\geq 0$, $y-5>0$, and $\ln(y-5)-6\neq 0$
$\Downarrow$ $\Downarrow$ $\Downarrow$
$x\geq 18$, $y>5$, and $\ln(y-5)\neq 6$
$y-5\neq e^6$
$y\neq e^6+5$

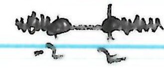
$$\boxed{\text{Domain: } \{(x,y) \mid x\geq 18,\ y>5,\ y\neq e^6+5\}}$$

all points in the plane subject to these conditions

Ex 3. Find the domain of $f(x,y) = \sqrt{16-x^2} + \sqrt{y^2-4}$

Need $16-x^2 \geq 0$ and $y^2-4 \geq 0$

$(4+x)(4-x) \geq 0$ and $(y+2)(y-2) \geq 0$



$-4 \leq x \leq 4$ $\quad$ $y \leq -2$ or $y \geq 2$

$|x| \leq 4$ $\quad$ $|y| \geq 2$

$$\boxed{\text{Domain: } \{(x,y) \mid |x| \leq 4 \text{ and } |y| \geq 2\}}$$

Ex 4. Find the domain of $f(x,y) = \dfrac{\ln(8-x-y)\sqrt{x+y-2}}{\sqrt[6]{x+y-4}}$

Need $8-x-y > 0$, $x+y-2 \geq 0$, $x+y-4 \gtrless 0$ (in denominator)

$\Downarrow$ $\quad$ $\Downarrow$ $\quad$ $\Downarrow$

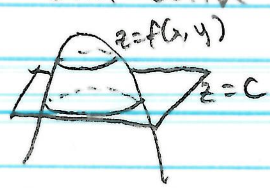
$8 > x+y$ $\quad$ $x+y \geq 2$ $\quad$ $x+y > 4$

all satisfied when $4 < x+y < 8$

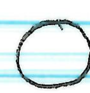
$$\boxed{\text{Domain: } \{(x,y) \mid 4 < x+y < 8\}}$$

Level Curves

Level curves are the curves you get by slicing a solid with a plane $z = c$



$\longrightarrow$ level curve



To find a level curve of $f(x,y)$ at $z=c$, set $c = f(x,y)$ and use algebra to get it into a form you recognize.

Ex 5. Identify the shapes of the level curves of $f(x,y) = 3\ln(2(x-5)^2 + 2(y+2)^2)$

$$C = 3\ln(2(x-5)^2 + 2(y+2)^2)$$
$$\frac{C}{3} = \ln(2(x-5)^2 + 2(y+2)^2)$$
$$e^{C/3} = 2(x-5)^2 + 2(y+2)^2$$
$$\frac{1}{2}e^{C/3} = (x-5)^2 + (y+2)^2$$

level curves at $z=C$ are circles centered at $(5,-2)$ with radius $\sqrt{\frac{1}{2}e^{C/3}}$

Ex 6. Identify the shape of the level curve of $f(x,y) = 16xy^2$ at $z=1$

$$1 = 16xy^2 \Rightarrow x = \frac{1}{16y^2}$$

This is a rational function with x-axis symmetry (since opposite y-values give the same x-value)

Ex 7. A store has two types of chocolate. If milk chocolate is sold for x dollars and dark chocolate is sold for y dollars, demand is

$$D_{milk}(x,y) = 300 + 2x - y$$
$$D_{dark}(x,y) = 400 + 3y - x$$

What is a function for the company's revenue for chocolate?

$$\text{Revenue} = \underbrace{(\#\text{ sold})}_{(\text{demand})}(\text{cost per item})$$

$$\boxed{x(300+2x-y) + y(400+3y-x)}$$