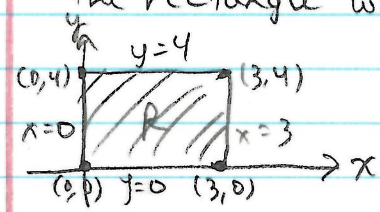
MA 16020

Lesson 28

Double Integrals (Part 2)

Sometimes you will need to set up a double integral or switch the order of integration. To do this, sketch the region and find the equations of all curves involved. The inner integral goes from curve to curve, and the outer integral goes from value to value.

Ex 1. Evaluate $\iint_R 8x^3y\, dA$ where R is the rectangle with vertices (0,0), (3,0), (0,4), (3,4).



2 approaches: dy dx or dx dy

dy dx: y varies from curve $y=0$ to curve $y=4$
x varies from value $x=0$ to value $x=3$

$$\int_0^3 \int_0^4 8x^3y\, dy\, dx$$

dx dy: x varies from curve $x=0$ to curve $x=3$
y varies from value $y=0$ to value $y=4$

$$\int_0^4 \int_0^3 8x^3y\, dx\, dy$$

$$= \int_0^4 2x^4y \Big|_0^3 dy = \int_0^4 162y\, dy = 81y^2\Big|_0^4$$

$$= \boxed{1296}$$

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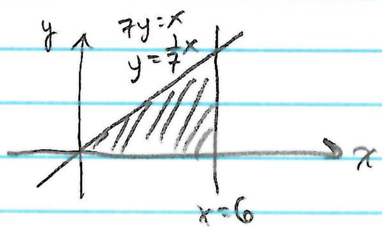
Ex 2. Set up a double integral representing representing the volume under the surface $f(x,y) = 5(x+y)$ over the region R bounded by $y = \frac{1}{7}x$, $x = 6$, and the x-axis.



dx dy : x varies from curve $x = 7y$ to curve $x = 6$

y varies from value $y = 0$ to value $y = \frac{6}{7}$

$$\boxed{\int_0^{6/7} \int_{7y}^{6} 5(x+y)\, dx\, dy}$$

dy dx : y varies from curve $y = 0$ to $y = \frac{1}{7}x$

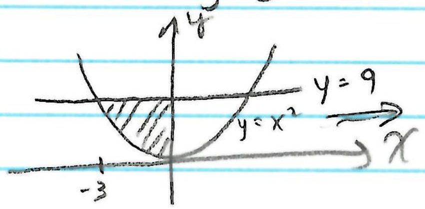
x varies from value $x = 0$ to value $x = 6$

$$\boxed{\int_0^6 \int_0^{\frac{1}{7}x} 5(x+y)\, dy\, dx}$$

Ex 3. Switch the order of integration

$$\int_{-3}^{0} \int_{x^2}^{9} f(x,y)\, dy\, dx$$

bounded by $y = x^2$ to $y = 9$ from $x = -3$ to $x = 0$



$\longrightarrow x = \pm\sqrt{y}$

left half : $-\sqrt{y}$

dx dy : x varies from curve $x = -\sqrt{y}$ to curve $x = 0$

y varies from value $y = 0$ to value $y = 9$

$$\boxed{\int_0^9 \int_{-\sqrt{y}}^{0} f(x,y)\, dx\, dy}$$

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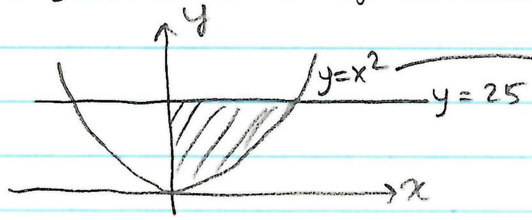
Ex 4. Compute $\int_0^5 \int_{x^2}^{25} -9x\sqrt{1+y^2}\,dy\,dx$ (Round to 2 decimals)

Cannot compute the inner integral, so switch order of integration

y varies from $y=x^2$ to $y=25$ from $x=0$ to $x=5$



$y=x^2 \rightarrow x=\pm\sqrt{y}$

right half $x=\sqrt{y}$

dx dy: x varies from curve $x=0$ to $x=\sqrt{y}$

y varies from value $y=0$ to $y=25$

$$\int_0^{25}\int_0^{\sqrt{y}} -9x\sqrt{1+y^2}\,dx\,dy$$

$$= \int_0^{25} -\frac{9}{2}x^2\sqrt{1+y^2}\Big|_0^{\sqrt{y}}\,dy$$

$$= \int_0^{25}\left(-\frac{9}{2}y\sqrt{1+y^2}+0\right)dy$$

$u=1+y^2$, $du=2y\,dy \Rightarrow \frac{du}{2}=y\,dy$

$u(0)=1+0^2=1$, $u(25)=1+25^2=626$

$$\int_1^{626} -\frac{9}{4}u^{1/2}\,du = -\frac{3}{2}u^{3/2}\Big|_1^{626}$$

$$= -\frac{3}{2}(626)^{3/2}+\frac{3}{2}(1)^{3/2}$$

$$\approx -23492.27$$