MA 16020

Lesson 31

Gauss-Jordan Elimination

Last time, we learned Gaussian elimination, which is a way to get a matrix into row-echelon form.

$\left(\left[\begin{array}{cc|c} 1 & a & b \\ 0 & 1 & c \end{array}\right] \text{ or } \left[\begin{array}{ccc|c} 1 & a & b & d \\ 0 & 1 & c & e \\ 0 & 0 & 1 & f \end{array}\right] \text{ etc.} \right.$

diagonal entries 1, below diagonal 0)

It is very easy to read off a solution if you get the matrix into reduced row-echelon form

$\left(\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right] \text{ or } \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right] \text{ etc.,} \right.$

diagonal entries 1, below and above diagonal 0)

Gauss-Jordan Elimination

To get an augmented matrix into reduced row-echelon form

1. Perform Gaussian Elimination to get it into row echelon form.
2. Use elementary row operations to turn entries above (3,3) into 0's and then above (2,2) into 0's. etc.

   Make use of the 1 in positions (3,3) and (2,2) to do this!

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Ex 1. Transform the augmented matrix into reduced row-echelon form

$$\left[\begin{array}{ccc|c} -1 & 2 & 2 & -1 \\ 3 & 0 & 2 & -1 \\ -2 & 1 & -1 & 2 \end{array}\right]$$

$$\xrightarrow{-R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 3 & 0 & 2 & -1 \\ -2 & 1 & -1 & 2 \end{array}\right] \xrightarrow{-3R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 6 & 8 & -4 \\ -2 & 1 & -1 & 2 \end{array}\right]$$

$$\xrightarrow{2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 6 & 8 & -4 \\ 0 & -3 & -5 & 4 \end{array}\right] \xrightarrow{\frac{1}{6}R_2} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{2}{3} \\ 0 & -3 & -5 & 4 \end{array}\right]$$

$$\xrightarrow{3R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{2}{3} \\ 0 & 0 & -1 & 2 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \text{(REF)}$$

$$\xrightarrow{-\frac{4}{3}R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{2R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{2R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]}$$

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Ex 2. Use Gauss-Jordan elimination to solve the system

$$\begin{cases} 3x + y - z = -1 \\ 2x - y + 2z = 4 \\ x - 2y + z = -1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & -1 \\ 2 & -1 & 2 & 4 \\ 1 & -2 & 1 & -1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & -1 & 2 & 4 \\ 3 & 1 & -1 & -1 \end{array}\right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & 0 & 6 \\ 3 & 1 & -1 & -1 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & 0 & 6 \\ 0 & 7 & -4 & 2 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 7 & -4 & 2 \end{array}\right]$$

$$\xrightarrow{-7R_2+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{-R_3+R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{2R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\boxed{(0, 2, 3)}$$