MA 16020
Lesson 5
Integration by Parts (Part 2)

Ex 1. Find the area of the region bounded by
$y = x^2\cos(3x) + 1,\ y = 0,\ x = 0,\ x = \pi$

$$\int_0^{\pi}(x^2\cos(3x)+1)\,dx = \underbrace{\int_0^{\pi} x^2\cos(3x)\,dx} + \int_0^{\pi} 1\,dx$$

By LATE, $u = x^2$ $\quad dv = \cos(3x)\,dx$
$du = 2x\,dx \quad v = \frac{1}{3}\sin(3x)$

$$\frac{1}{3}x^2\sin(3x) - \frac{2}{3}\underbrace{\int x\sin(3x)\,dx}$$

$$\int x\sin(3x)\,dx$$

$u = x \quad dv = \sin(3x)\,dx$
$du = dx \quad v = -\frac{1}{3}\cos(3x)$

$$-\frac{1}{3}x\cos(3x) + \frac{1}{3}\underbrace{\int \cos(3x)\,dx}_{\frac{1}{3}\sin(3x)}$$

$$\frac{1}{3}x^2\sin(3x) - \frac{2}{3}\left[-\frac{1}{3}x\cos(3x) + \frac{1}{3}\left(\frac{1}{3}\sin(3x)\right)\right]$$

$$\left(\frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x)\right)\Big|_0^{\pi}$$

$$\left(\frac{1}{3}\pi^2\sin(3\pi) + \frac{2}{9}\pi\cos(3\pi) - \frac{2}{27}\sin(3\pi)\right) - (0 + 0 - 0)$$

$$0 + -\frac{2}{9}\pi - 0) - 0$$

$$\int_0^{\pi} x^2\cos(3x)\,dx = -\frac{2}{9}\pi$$

Also $\int_0^{\pi} 1\,dx = x\Big|_0^{\pi} = \pi - 0 = \pi$

$$\pi - \frac{2}{9}\pi = \boxed{\frac{7\pi}{9}}$$

Ex 2. A tree grows at a rate of $\frac{2\ln\sqrt{t+2}}{(t+2)^2}$ feet per year t years after it is planted. If the tree was 4 feet tall when it was planted, how tall will it be in 6 years? Round to nearest hundredth.

$$h(t) = \int \frac{2\ln\sqrt{t+2}}{(t+2)^2}\,dt = \int 2\ln\sqrt{t+2} \cdot (t+2)^{-2}\,dt$$

$$u = 2\ln\sqrt{t+2} \qquad dv = (t+2)^{-2}\,dt$$

$$u = 2\cdot\tfrac{1}{2}\ln(t+2) \qquad v = -(t+2)^{-1}$$

$$du = \frac{1}{t+2}\,dt$$

$$\frac{-2\ln\sqrt{t+2}}{t+2} - \int -\frac{1}{t+2}\cdot\frac{1}{t+2}\,dt$$

$$= \frac{-2\ln\sqrt{t+2}}{t+2} + \int (t+2)^{-2}\,dt$$

$$= \frac{-2\ln\sqrt{t+2}}{t+2} - \frac{1}{(t+2)} + C$$

$$4 = h(0) = -\ln\sqrt{2} - \tfrac{1}{2} + C, \text{ so } C = 4 + \ln\sqrt{2} + \tfrac{1}{2}$$

$$h(t) = \frac{-2\ln\sqrt{t+2}}{t+2} - \frac{1}{t+2} + 4 + \ln\sqrt{2} + \frac{1}{2}$$

$$h(6) = -\frac{2\ln\sqrt{8}}{8} - \frac{1}{8} + 4 + \ln\sqrt{2} + \frac{1}{2}$$

$$\approx \boxed{4.46 \text{ feet}}$$

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Ex 3. When samples of iron ore are tested for potential mining sites, the probability of finding a sample that has x percentage iron in the sample is described by $\frac{163}{100}\left(\frac{x}{\sqrt{1+6x}}\right)$, where x is between 0 and 1.

Find the probability that a tested sample is at least 82% iron. Round to 4 decimal places.

Want any probability from .82 to 1.

$$\int_{.82}^{1} \frac{163}{100} \frac{x}{\sqrt{1+6x}}\,dx = \frac{163}{100}\int_{.82}^{1} x(1+6x)^{-1/2}\,dx$$

$u = x \qquad dv = (1+6x)^{-1/2}dx$

$du = dx \qquad v = \frac{1}{3}(1+6x)^{1/2}$

$$\frac{163}{100}\left[\frac{1}{3}x(1+6x)^{\frac{1}{2}} - \int \frac{1}{3}(1+6x)^{1/2}dx\right]$$

$$= \frac{163}{100}\left[\frac{1}{3}x(1+6x)^{1/2} - \frac{1}{3}\cdot\frac{1}{6}\cdot\frac{2}{3}(1+6x)^{3/2}\right]$$

$$= \frac{163}{100}\left[\frac{1}{3}x(1+6x)^{1/2} - \frac{1}{27}(1+6x)^{3/2}\right]_{.82}^{1}$$

$$= \frac{163}{100}\left(\frac{1}{3}(7)^{1/2} - \frac{1}{27}(7)^{3/2} - \frac{1}{3}(.82)(1+6\cdot .82)^{1/2} + \frac{1}{27}(1+6\cdot .82)^{3/2}\right)$$

$$\approx \boxed{0.1050}$$

or

u-sub $\quad u = 1+6x, \; du = 6\,dx, \; \frac{1}{6}du = dx$

$\frac{1}{6}u - \frac{1}{6} = x$

$$\frac{163}{100}\cdot\frac{1}{6}\int_{5.92}^{7}\left(\frac{1}{6}u - \frac{1}{6}\right)u^{-1/2}\,du$$

$$\frac{163}{600}\int_{5.92}^{7} \frac{1}{6}u^{1/2} - \frac{1}{6}u^{-1/2}\,du$$

$$= \frac{163}{600}\left[\frac{1}{9}u^{3/2} - \frac{1}{3}u^{1/2}\right]_{5.92}^{7}$$

$$= \frac{163}{600}\left(\frac{1}{9}(7)^{3/2} - \frac{1}{3}(7)^{1/2} - \frac{1}{9}(5.92)^{3/2} + \frac{1}{3}(5.92)^{1/2}\right)$$

$$\approx \boxed{0.1050}$$