MA 16020

Lesson 7

Separable Equations (Part 1)

A diff eq is called separable if you can separate the variables to different sides of the equation.

To solve a separable equation, get all the y's on one side and all of the t's/x's on the other. Then integrate. Then solve for y.

Recall: A general solution has a "C" in it. A particular solution does not.

Ex 1. $\frac{dy}{dt} + t^k y = 0$, $y(0) = 1$, $y(1) = e^{-5}$.

Find k and y(t).

$$\frac{dy}{dt} = -t^k y$$

$$\frac{1}{y}\,dy = -t^k\,dt \longrightarrow \int \frac{1}{y}\,dy = \int -t^k\,dt$$

$$\ln|y| = -\frac{1}{k+1}t^{k+1} + C$$

$$y = Ce^{\left(-\frac{t^{k+1}}{k+1}\right)}$$

$$1 = y(0) = Ce^{\left(-\frac{0^{k+1}}{k+1}\right)} = Ce^0 = C$$

$$y = e^{\left(-\frac{t^{k+1}}{k+1}\right)}$$

$$e^{-5} = y(1) = e^{-\frac{1}{k+1}1^{k+1}} = e^{-1/k+1}$$

$$-5 = -\frac{1}{k+1}$$

$$\frac{1}{5} = k+1 \quad \text{so } \boxed{k = \frac{1}{5} - 1 = -\frac{4}{5}}$$

$$y(t) = e^{\left(-\frac{t^{-\frac{4}{5}+1}}{-\frac{4}{5}+1}\right)} = e^{\left(\frac{-t^{\frac{1}{5}}}{\left(\frac{1}{5}\right)}\right)} = \boxed{e^{-5t^{1/5}}}$$

MA 16020

Lesson 7

Ex 2. Find the particular solution $\frac{dy}{dt} + y\cos t = 0, \ y(\frac{\pi}{2}) = 1$

$$\frac{dy}{dt} = -y\cos t$$
$$\frac{1}{y}\,dy = -\cos t\,dt$$
$$\ln|y| = -\sin t + C$$
$$y = Ce^{-\sin t}$$
$$1 = Ce^{-1} \Rightarrow C = e$$
$$y = e \cdot e^{-\sin t} = \boxed{e^{-\sin t + 1}}$$

Ex 3. Find the general solution

$$\frac{dy}{dx} = \frac{3x-1}{5y^4}$$
$$5y^4\,dy = (3x-1)\,dx$$
$$y^5 = \frac{3}{2}x^2 - x + C$$
$$\boxed{y = (\frac{3}{2}x^2 - x + C)^{1/5}}$$

Ex 4. Find the general solution

$$\frac{dy}{dx} = 9x^2(3-y)$$
$$\frac{1}{3-y}\,dy = 9x^2\,dx$$
$$-\ln|3-y| = 3x^3 + C$$
$$\ln|3-y| = -3x^3 + C$$
$$|3-y| = Ce^{-3x^3}$$
$$3-y = Ce^{-3x^3}$$
$$-y = Ce^{-3x^3} - 3$$
$$\boxed{y = Ce^{-3x^3} + 3}$$

Ex 5. A towel hung on a clothesline to dry loses moisture at a rate proportional to its moisture content. After 1 hour, it has lost 38% of its original moisture content. After how long will the towel have lost 60% of its moisture content? (Round to 2 decimal places)

$$\frac{dM}{dt} = kM, \text{ so } M(t) = Ce^{kt}$$

At $t=0$, it has $100\% = 1.00$ moisture content

$$1 = Ce^0 = C$$

$$M(t) = e^{kt}$$

At $t=1$, it has lost 38% of moisture, so it has $100\% - 38\% = 62\% = 0.62$ left

$$0.62 = e^{k(1)}, \text{ so } k = \ln(0.62)$$

$$M(t) = e^{\ln(0.62)t}$$

When it has lost 60%, it has $100\% - 60\% = 40\% = 0.4$

$$0.4 = e^{\ln(0.62)t}$$

$$\ln(0.4) = \ln(0.62)t$$

$$t = \frac{\ln(0.4)}{\ln(0.62)} \approx \boxed{1.92 \text{ hours}}$$