Eddie Price **Quiz 11 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 1; Average score: 7.94
Letter grade estimates: **A: 10, B: 9, C: 7-8, D: 4-6, F: 0-3**

Problem 1 (4 Points). The pressure P of an ideal gas is related to its volume V and its temperature T by the equation $PV = 0.4T$. The temperature is measured with an error of 7 Kelvin, and the volume is measured with an error of 0.3 m^3. If it is known that the actual values are $T = 243$ Kelvin and $V = 4$ m^3, what is the estimated maximum error in the measurement of the pressure? (Round to 4 decimal places.)

Solution. For a variable x, the error in measuring x can be seen as $\Delta x = x_{\text{actual}} - x_{\text{measured}}$. As such, to find the maximum error in the measurement of pressure, we seek ΔP. To do this, we solve for P and get $P = 0.4TV^{-1}$. Using our formula for differentials, we know that $\Delta P \approx \frac{\partial P}{\partial T}\Delta T + \frac{\partial P}{\partial V}\Delta V$. We can see from the problem that $\Delta T = \pm 7$ and $\Delta V = \pm 0.3$ are the maximum errors and that the actual values are $T = 243$ and $V = 4$. Taking partial derivatives, we get $\frac{\partial P}{\partial T} = 0.4V^{-1} = \frac{0.4}{V}$ and $\frac{\partial P}{\partial V} = -0.4TV^{-2} = \frac{-0.4T}{V^2}$. Plugging this into the formula for ΔP and evaluating the partial derivatives at the actual values, we get:

$$\Delta P \approx \left(\frac{0.4}{V}\right) \cdot (\Delta T) + \left(\frac{-0.4T}{V^2}\right) \cdot (\Delta V)$$

$$\Delta P \approx \frac{0.4}{(4)} \cdot (\pm 7) + \frac{-0.4\,(243)}{(4)^2} \cdot (\pm 0.3) \approx \pm 1.1225 \text{ or } \pm 2.5225$$

As we can see, the maximum error is therefore $\Delta P \approx \boxed{\pm 2.5225}$ kPa.

Problem 2 (5 points). The volume V of a cylindrical can with radius r and height h is given by $V(r, h) = \pi r^2 h$. A particular can is 12 cm tall and has a radius of 3 cm. If the height is increased by 1.4 cm, use differentials to estimate the change in radius needed so that the volume stays the same. (Round your answer to 4 decimal places.) Make sure to specify whether the radius needs to increase or decrease.

Solution. Since V is a function of r and h, we get $\Delta V \approx \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h$. From the problem, we know $\Delta h = 1.4$ and we want $\Delta V = 0$. We seek Δr to make this happen. We must find the partial derivatives: $\frac{\partial V}{\partial r} = 2\pi r h$ and $\frac{\partial V}{\partial h} = \pi r^2$. The current values are $r = 3$ and $h = 12$, so plugging these into the formula for ΔV, we get

$$\Delta V \approx (2\pi r h) \cdot (\Delta r) + \left(\pi r^2\right) \cdot (\Delta h)$$

$$0 \approx (2\pi \cdot 3 \cdot 12) \cdot \Delta r + \left(\pi \cdot 3^2\right) \cdot (1.4)$$

Solving for Δr, we get

$$\Delta r \approx \frac{-\pi \cdot 3^2 \cdot 1.4}{2\pi \cdot 3 \cdot 12} \approx -0.1750$$

So the radius needs to $\boxed{\text{decrease approximately 0.1750 cm}}$.

Common Mistakes

No particularly common mistakes on this one.