Eddie Price **Quiz 12 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 1; Average score: 6.41
Letter grade estimates: **A: 10, B+: 9, B: 8, B-: 7, C+: 6, C: 4-5, C-: 3, D: 2, F: 0-1**

Problem 1 (9 Points). Find and classify the critical points (u, v) of the function

$$g(u, v) = 6u^2v - 24uv - v^2$$

Solution. We must first find the critical points:

$$g_u = 12uv - 24v \overset{\text{set}}{=} 0, \qquad g_v = 6u^2 - 24u - 2v \overset{\text{set}}{=} 0$$

For the first equation ($g_u = 0$), we can factor out $12v$ to get $12v\,(u-2) = 0$. We conclude that $v = 0$ or $u = 2$. So we see that in order for $g_u = 0$ to be true, either $v = 0$ must be true or $u = 2$ must be true. We explore these two cases to make $g_v = 0$ true.

Case $v = 0$: Plugging $v = 0$ into the second equation ($g_v = 0$), we get $6u^2 - 24u = 0$, which can be factored as $6u\,(u-4) = 0$, from which we conclude that $u = 0$ or $u = 4$. Thus, we get the two critical points (of the form (u, v)) $(0, 0)$ and $(4, 0)$.

Case $u = 2$: Plugging $u = 2$ into the second equation, we get $6(2)^2 - 24\,(2) - 2v = 0$, or $-24 - 2v = 0$. Hence, we conclude (in this case) $v = -12$. Thus, we get the critical point (in the form (u, v)) $(2, -12)$.

Next, to find the discriminant function $D(u, v) = g_{uu}g_{vv} - (g_{uv})^2$, we must find the second-order partial derivatives:

$$g_{uu} = 12v, \qquad g_{vv} = -2, \qquad g_{uv} = 12u - 24$$

So $D(u, v) = (12v)\,(-2) - (12u - 24)^2$, or $D(u, v) = -24v - (12u - 24)^2$.

Critical point $(0, 0)$: $D(0, 0) = 0 - (-24)^2 < 0$, so $\boxed{g \text{ has a saddle point at } (0,0)}$.

Critical point $(4, 0)$: $D(4, 0) = 0 - (12 \cdot 4 - 24)^2 < 0$, so $\boxed{g \text{ has a saddle point at } (4,0)}$.

Critical point $(2, -12)$: $D(2, -12) = -24\,(-12) - (12 \cdot 2 - 24)^2 > 0$, so we must check $g_{uu}(2, -12)$ or $g_{vv}(2, -12)$. We see that these are negative. Hence, $\boxed{g \text{ has a relative maximum at } (2,-12)}$.

Common Mistakes

The most common mistakes came from finding the critical points. Some people divided by a variable and lost some critical points because of that. Some people factored the first equation, got $v = 0$ or $u = 2$ and thought that meant $(2, 0)$ was a solution (which is not true since a critical point (a, b) makes both $g_u(a, b) = 0$ and $g_v(a, b) = 0$ at the same time).

Some people also had mistakes applying the second derivative test.