Eddie Price **Quiz 13 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 1; Average score: 6.67
Letter grade estimates: **A: 10, B+: 9, B: 8, C+: 7, C: 6, C-: 5, D: 3-4, F: 0-2**

Problem 1 (4 Points). What is the minimum value of $f(x,y) = x^2e^{y^2}$ subject to the constraint $20y^2 + x = 10$?

Solution. $f_x = 2xe^{y^2}$, $f_y = 2x^2ye^{y^2}$, $g_x = 1$, and $g_y = 40y$. So we get the following system of equations:

$$2xe^{y^2} = \lambda, \qquad 2x^2ye^{y^2} = \lambda 40y, \qquad 20y^2 + x = 10$$

The 1st equation is solved for λ, so plugging into the 2nd equation, $2x^2ye^{y^2} = \left(2xe^{y^2}\right)40y$. After simplifying, subtracting things from both sides, and factoring, we get $2xye^{y^2}(x-40) = 0$. As such, we have three possibilities: $x = 0$, $y = 0$, or $x = 40$.
Case $x = 0$: We plug into the 3rd equation to get $20y^2 = 10$, or $y^2 = \frac{1}{2}$, or $y = \pm\sqrt{\frac{1}{2}}$. So we get the two points $\left(0, \sqrt{\frac{1}{2}}\right)$ and $\left(0, -\sqrt{\frac{1}{2}}\right)$
Case $y = 0$: We plug into the 3rd equation to get $x = 10$. So we get the point $(10, 0)$.
Case $x = 40$: We plug into the 3rd equation to get $20y^2 = -30$, which has no real solutions. So we obtain no points to check.

Plugging these into f, we get $f\left(0, \sqrt{\frac{1}{2}}\right) = 0$, $f\left(0, -\sqrt{\frac{1}{2}}\right) = 0$, and $f(10, 0) = 100$. So the minimum value of f subject to the constraint is $\boxed{0}$.

Problem 2 (5 Points). A company's total profit from selling x thousand units of Product A and y thousand units of Product B is $P(x,y) = 5.1x + 6.9y$ measured in millions. The quantities produced must satisfy the production possibilities curve $x^2 + y^2 = 144$. Assuming a maximum exists, how many units of each product should the company produce so that profit is maximized? (Round your answers to the nearest integer.) Clearly label which product has which amount.

Solution. $P_x = 5.1$, $P_y = 6.9$, $g_x = 2x$, $g_y = 2y$. So we get the following system of equations:

$$5.1 = \lambda 2x, \qquad 6.9 = \lambda 2y, \qquad x^2 + y^2 = 144$$

Solving the 1st equation for λ, we obtain $\lambda = \frac{5.1}{2x}$, and plugging into the 2nd equation, we get $6.9 = \left(\frac{5.1}{2x}\right)2y$. We can solve for x by multiplying both sides by x and dividing both sides by 6.9 to get $x = \frac{5.1}{6.9}y$. Plugging this into the constraint, we have $\left(\frac{5.1}{6.9}y\right)^2 + y^2 = 144$, so $\left(1 + \left(\frac{5.1}{6.9}\right)^2\right)y^2 = 144$. In other words, $y^2 = 144/\left(1 + \left(\frac{5.1}{6.9}\right)^2\right)$, giving $y \approx 9.650$ (since the negative option doesn't make sense). Plugging back into $x = \frac{5.1}{6.9}y$, we get $x \approx 7.133$.

Since x and y are measured in thousands, $\boxed{\text{Product A: } 7{,}133; \text{Product B: } 9{,}650}$.

<u>Common Mistakes</u>

For problem 1, many people cancelled variables which caused them to lose points which they needed to check.

For problem 1, many people found where the minimum value occurs instead of finding the minimum value itself.

For problem 2, many people made algebra errors in solving the system of equations.

For problem 2, many people found the maximum profit instead of the amount of each product needed to maximize the profit.

For problem 2, many people forgot to multiply the values of x and y by 1000 to get the amount of each product needed. The rounding should occur after this, not before this.