**Eddie Price** 

High score: 10; (nonzero) Low score: 1; Average score: 6.32 Letter grade estimates: A: 10, B: 9, B-: 8, C+: 7, C: 5-6, C-: 4, D: 2-3, F: 0-1

<u>Problem 1</u> (3 Points). Evaluate the double integral

$$\int_{-5}^{5} \int_{0}^{5} \left( 6x + 10y \right) \, dy \, dx$$

<u>Solution</u>. Evaluate the inner integral first:

$$\int_{-5}^{5} (6xy + 5y^2) \Big|_{y=0}^{y=5} dx = \int_{-5}^{5} (30x + 125) dx$$
$$= (15x^2 + 125x) \Big|_{-5}^{5} = 1000 - (-250) = \boxed{1250}$$

<u>Problem 2</u> (3 Points). Evaluate the double integral

$$\int_{5\pi}^{8\pi} \int_{0}^{y} -6\csc(y)\cos(x) \, dx \, dy$$

Solution. Evaluate the inner integral first:

$$\int_{5\pi}^{8\pi} -6\csc(y)\sin(x) \mid_{x=0}^{x=y} dy = \int_{5\pi}^{8\pi} (-6\csc(y)\sin(y) + 6\csc(y)\sin(0)) dy$$
$$= \int_{5\pi}^{8\pi} -6 dy \quad (\text{since } \csc(y)\sin(y) = 1 \text{ and } \sin(0) = 0)$$
$$= -6y \mid_{5\pi}^{8\pi} = -6(8\pi) + 6(5\pi) = \boxed{-18\pi}$$

<u>Problem 3</u> (3 Points). Evaluate the double integral, and round to 2 decimal places.

$$\int_{1}^{e} \int_{0}^{3\ln(x)} 3x \, dy \, dx$$

<u>Solution</u>. Evaluate the inner integral first:

$$\int_{1}^{e} \int_{0}^{3\ln(x)} 3xy \mid_{y=0}^{y=3\ln(x)} dx = \int_{1}^{e} 9x\ln(x) dx$$

Use integration by parts. By LATE, we pick  $u = \ln(x)$  (so  $du = \frac{1}{x} dx$ ) and dv = 9x dx, so  $v = \frac{9}{2}x^2$ . The formula is given by  $\int u dv = uv - \int v du$ , so we have:

$$\int 9x \ln(x) \, dx = \frac{9}{2}x^2 \ln(x) - \int \frac{9}{2}x^2 \cdot \frac{1}{x} \, dx = \frac{9}{2}x^2 \ln(x) - \frac{9}{4}x^2$$

Now evaluate the integral from x = 1 to x = e:

$$\left[\frac{9}{2}(e)^2\ln(e) - \frac{9}{4}e^2\right] - \left[\frac{9}{2}(1)^2\ln(1) - \frac{9}{4}(1)^2\right] \approx \boxed{18.88}$$

Spring 2018

## Common Mistakes

For problem 1, many people did not integrate 6x correctly with respect to y. Since 6x is treated as a constant when integrating with respect to y, the antiderivative of 6x with respect to y is 6xy.

For problem 2, many people got the sign incorrect when integrating  $\cos(x)$ .

For problem 2, many people didn't notice that  $\csc(y)\sin(y) = \frac{1}{\sin(y)}\sin(y) = 1$ .

For problem 3, some people integrated the inner integral with respect to x, when it should be integrated with respect to y.

For problem 3, many people didn't use integrating by parts when you need to.

For problem 3, many people didn't remember the formula for integration by parts correctly.