Eddie Price **Quiz 1 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 4; Average score: 8.04

Problem 1 (5 Points). Evaluate the definite integral. Give the *exact* answer (do not round).

$$\int_0^{\pi/3} \sec x\,(4\sec x + 3\tan x)\,dx$$

Solution. First, distribute $\sec x$ to both terms, then recognize that both terms are derivatives of functions you know:

$$\int_0^{\pi/3} \left(4\sec^2 x + 3\sec x\tan x\right)dx$$

$$(4\tan x + 3\sec x)\big|_0^{\pi/3}$$

$$\left[4\tan\left(\frac{\pi}{3}\right) + 3\sec\left(\frac{\pi}{3}\right)\right] - [4\tan(0) + 3\sec(0)]$$

$$\left[4\left(\sqrt{3}\right) + 3\,(2)\right] - [4\,(0) + 3\,(1)]$$

$$\boxed{4\sqrt{3}+3}$$

Problem 2 (4 points). Evaluate the indefinite integral.

$$\int \frac{1}{\sqrt{x}}\cos\left(3+\sqrt{x}\right)dx$$

Solution. Notice that this looks like the result of the chain rule, so we use substitution. Notice that the innermost function is $3+\sqrt{x}$, so we set

$$u = 3+\sqrt{x} = 3 + x^{1/2}$$

Hence, $\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$. Thus, $2du = \frac{1}{\sqrt{x}}dx$.
We can rewrite our integral as

$$\int \cos\left(3+\sqrt{x}\right)\cdot\frac{1}{\sqrt{x}}dx$$

Substituting $u = 3+\sqrt{x}$ and $2du = \frac{1}{\sqrt{x}}dx$, we obtain the new integral

$$2\int \cos(u)\,du$$

$$2\sin(u) + C$$

Since $u = 3+\sqrt{x}$, we have

$$\boxed{2\sin\left(3+\sqrt{x}\right)+C}$$

Common Mistakes

For problem 1, many people forgot their unit circle values.

For problem 1, many people tried to use substitution when it was unnecessary and did not work.

For problem 1, some people included a "$+C$" in their answer. This is incorrect since a definite integral has a definite value (a definite integral represents the area under a curve, which is a specific, explicit number). The $+C$ for indefinite integrals comes from the fact that an indefinite integral represents antidifferentiating. Each function $f(x)$ has infinitely many antiderivatives, but they can be shifted vertically from one another, so they only differ by a constant.

For problem 1, many people rounded instead of giving an exact answer.

For problem 2, many people forgot the "$+C$" in their answer. This is an indefinite integral, meaning we must include $+C$. (See lesson R.)

For problem 2, people confused the signs for integrating $\cos u$. Remember that the derivative of $\sin u$ is $\cos u$; thus, the integral of $\cos u$ is $\sin u + C$.

For both problems, many people made algebra errors.