Eddie Price **Quiz 2 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 2; Average score: 7.14

Problem 1 (9 Points). The oscillating current in an electrical circuit is $I = 6\sin(3t) + 8\cos(2t)$, where I is measured in amperes and t is measured in seconds. Set up an integral and find the average current for the time interval $0 \le t \le \pi$. Round to 3 decimal places.

Solution. The average value of a function $f(x)$ on $a \le x \le b$ is $\frac{1}{b-a}\int_a^b f(x)\ dx$. As such, the average oscillating current is

$$\boxed{\frac{1}{\pi}\int_0^\pi (6\sin(3t) + 8\cos(2t))\,dt}$$

The arguments of the trig functions make it a bit tricky because of the chain rule. But the two trig functions have different arguments. Split up the integral into two separate integrals:

$$\frac{1}{\pi}\int_0^\pi (6\sin(3t) + 8\cos(2t))\,dt = \frac{1}{\pi}\left[\int_0^\pi 6\sin(3t)\ dt + \int_0^\pi 8\cos(2t)\ dt\right]$$

For the first integral, $\int_0^\pi 6\sin(3t)\ dt$, make the substitution $u = 3t$. Then $du = 3dt$, or $dt = \frac{1}{3}du$.

$$\int_{t=0}^{t=\pi} \frac{6}{3}\sin(u)\ du$$
$$-2\cos(u)|_{t=0}^{t=\pi}$$
$$-2\cos(3t)|_0^\pi$$
$$-2\cos(3\pi) + 2\cos(0)$$
$$2 + 2 = 4$$

For the second integral, $\int_0^\pi 8\cos(2t)\ dt$, make the substitution $u = 2t$. Then $du = 2dt$, or $dt = \frac{1}{2}du$.

$$\int_{t=0}^{t=\pi} \frac{8}{2}\cos(u)\ du$$
$$4\sin(u)|_{t=0}^{t=\pi}$$
$$4\sin(2t)|_0^\pi$$
$$4\sin(2\pi) - 4\sin(0)$$
$$0 - 0 = 0$$

Thus, the average value is

$$\frac{1}{\pi}\left[\int_0^\pi 6\sin(3t)\ dt + \int_0^\pi 8\cos(2t)\ dt\right] = \frac{1}{\pi}[4+0] = \frac{4}{\pi}$$

Rounded to 3 decimal places,

$$\boxed{1.273 \text{ amperes}}$$

<u>Common Mistakes</u>

Many people made a wrong substitution.

Many people did not make a substitution and did not adjust the coefficients accordingly.

Many people confused the signs for integrating $\cos u$ and $\sin u$. Remember that the derivative of $\sin u$ is $\cos u$; thus, the integral of $\cos u$ is $\sin u + C$. Similarly, the derivative of $-\cos u$ is $\sin u$; thus, the integral of $\sin u$ is $-\cos u + C$.

Many people forgot the $\frac{1}{\pi}$.

Many people did not round, even though the problem asked to.