Eddie Price **Quiz 3 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 1; Average score: 7.21

Problem 1 (4 Points). Evaluate the indefinite integral. (Use C as an arbitrary constant.)

$$\int \tan(x)\ dx$$

Solution. Notice that $\tan(x) = \frac{\sin(x)}{\cos(x)}$, so we have the integral:

$$\int \frac{\sin(x)}{\cos(x)}\ dx$$

Use substitution: Let $u = \cos(x)$. Then $du = -\sin(x)\ dx$, so $-du = \sin(x)\ dx$. Substituting into the original integral, we get

$$-\int \frac{1}{u}\ du$$

$$-\ln|u| + C$$

$$\boxed{-\ln|\cos(x)| + C}$$

Problem 2 (5 points). Evaluate the indefinite integral. (Use C as an arbitrary constant.)

$$\int x\ln(x)\ dx$$

Solution. For this problem, we must use integration by parts. By LATE, we should make the following selections:

$$u = \ln(x) \qquad dv = x\,dx$$

$$du = \tfrac{1}{x}\,dx \qquad v = \tfrac{1}{2}x^2$$

Plugging into $uv - \int v\,du$, we get

$$\frac{1}{2}x^2\ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\ dx$$

$$\frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x\,dx$$

$$\frac{1}{2}x^2\ln(x) - \frac{1}{2}\cdot\frac{1}{2}x^2 + C$$

$$\boxed{\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C}$$

<u>Common Mistakes</u>

Many students didn't even know how to start the problems. Some people tried to use integration by parts on problem 1 (which doesn't work) or use u-substitution on problem 2 (which doesn't work).

For problem 1, many students differentiated $\cos(x)$ wrong, by leaving out the negative sign. Remember, the *derivative* of $\cos(x)$ is $-\sin(x)$, but the *integral* of $\cos(x)$ is $\sin(x) + C$.

For problem 1, many students forgot the absolute values when integrating. $\int \frac{1}{u} du = \ln|u| + C$.

For problem 2, many students made the wrong choice for u and dv. We never discussed how to integrate $\ln(x)$, so it doesn't make much sense to choose $u = x$ and $dv = \ln(x)\ dx$. A lot of the time when students made this choice, they said $v = \frac{1}{x}$. Remember that $\frac{1}{x}$ is the *derivative* of $\ln(x)$, not the integral, so this is wrong. The integral of $\ln(x)$ is $x\ln(x) - x + C$ (use integration by parts to see this with $u = \ln(x)$ and $dv = dx$).