Eddie Price **Quiz 4 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 2; Average score: 8.53

Problem 1 (4 Points). Find the general solution to the differential equation. Make your arbitrary constant C as simple as possible.

$$\frac{dy}{dx} = \frac{x + x^{-1}}{y^2}$$

Solution. Separate variables by multiplying both sides by y^2 and dx, and then integrate.

$$\int y^2\, dy = \int \left(x + \frac{1}{x}\right)\, dx$$

$$\frac{1}{3}y^3 = \frac{1}{2}x^2 + \ln|x| + C$$

To solve for y, we multiply both sides by 3.

$$y^3 = \frac{3}{2}x^2 + 3\ln|x| + 3C$$

Now, take the cube root (or 1/3 power) of both sides. Since $3C$ is still an arbitrary constant, we can replace $3C$ by C.

$$\boxed{y = \sqrt[3]{\frac{3}{2}x^2 + 3\ln|x| + C} = \left(\frac{3}{2}x^2 + 3\ln|x| + C\right)^{1/3}}$$

Problem 2 (5 points). A bacteria culture grows at a rate directly proportional to its population. If the population is 5000 at time $t = 0$ and 8000 at $t = 1$, find the population at time $t = 3$. Round to the nearest whole number.

Solution. Let $P = P(t)$ represent the population of the bacteria at time t. Since P grows at a rate directly proportional to P, we have the following differential equation: $\frac{dP}{dt} = kP$ for some constant k. Separating variables and solving the differential equation gives us

$$\int \frac{1}{P}\, dP = \int k\, dt$$

$$\ln|P| = kt + C$$

$$|P| = e^{kt+C} = e^C e^{kt} \text{ “} = \text{” } Ce^{kt}$$

$$P(t) = \pm Ce^{kt} \text{ “} = \text{” } Ce^{kt}$$

We use $P(t) = Ce^{kt}$. We have $5000 = P(0) = Ce^0 = C$, giving $P(t) = 5000e^{kt}$. Also, $8000 = P(1) = 5000e^k$, so $\frac{8}{5} = e^k$; hence, $k = \ln\left(\frac{8}{5}\right)$. This gives the equation

$$P(t) = 5000e^{\ln\left(\frac{8}{5}\right)t}$$

$$P(3) = 5000e^{\ln\left(\frac{8}{5}\right)\cdot 3} = \boxed{20,480 \text{ bacteria}}$$

<u>Common Mistakes</u>

For problem 1, many people ended up with $\int \frac{1}{y^2}\, dy$ on the left hand side, when it should be $\int y^2\, dy$.

For problem 1, many people integrated x^{-1} incorrectly. Notice that $\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \ln|x| + C$.

For problem 1, you had $\frac{1}{3}y^3$ on the left hand side. To solve for y, you need to multiply by 3 first, and then take the cube root. The 3 is under the cube root.

For problem 2, some people set up the differential equation incorrectly.