Eddie Price **Quiz 5 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 5; Average score: 8.67

Problem 1 (4 Points). A 1000-gallon tank initially contains 700 gallons of brine containing 75 pounds of dissolved salt. Brine containing 2 pounds of salt per gallon ows into the tank at the rate of 5 gallons per minute, and the well-stirred mixture flows out of the tank at the rate of 1 gallon per minute. Set up a differential equation for the amount of salt, $A(t)$, in the tank at time t.

Solution. We know rates of flow here, so we get

$$\frac{dA}{dt} = (\text{rate of amount of salt flowing in}) - (\text{rate of amount of salt flowing out})$$

$$\frac{dA}{dt} = (\text{concentration in})\,(\text{rate in}) - (\text{concentration out})\,(\text{rate out})$$

$$\frac{dA}{dt} = \left(\frac{2 \text{ lbs}}{\text{gal}}\right)\left(\frac{5 \text{ gal}}{\text{min}}\right) - \left(\frac{\text{amount in tank (lbs) at time } t}{\text{volume in the tank (gal) at time } t}\right)\left(\frac{1 \text{ gal}}{\text{min}}\right)$$

The amount in the tank at time t is simply $A(t) = A$. For the volume, we know that it starts with 700 gallons, and that every minute 5 gallons flow in and 1 gallon flows out. As such, the tank *gains* $5 - 1 = 4$ gallons per minute. Thus, the volume is $V(t) = 700 + 4t$

$$\frac{dA}{dt} = \left(\frac{2 \text{ lbs}}{\text{gal}}\right)\left(\frac{5 \text{ gal}}{\text{min}}\right) - \left(\frac{A \text{ lbs}}{700 + 4t \text{ gal}}\right)\left(\frac{1 \text{ gal}}{\text{min}}\right)$$

$$\boxed{\frac{dA}{dt} = 10 - \frac{A}{700 + 4t}}$$

Problem 2 (5 points). Find the particular solution to the initial value problem

$$\frac{dy}{dx} = 3x^2\,(7 + y)\,, \qquad y(0) = 10$$

Solution. Separate variables then integrate:

$$\int \frac{1}{7+y}\,dy = \int 3x^2\,dx$$

$$\ln|7 + y| = x^3 + C$$

$$7 + y = Ce^{x^3}$$

$$y = Ce^{x^3} - 7$$

Use $y(0) = 10$ to find C:

$$10 = y(0) = Ce^{0^3} - 7 = Ce^0 - 7 = C - 7$$

So $C = 10 + 7 = 17$.

$$\boxed{y = 17e^{x^3} - 7}$$

Common Mistakes

For problem 1, many people got the concentration of the salt in the tank wrong. In particular, there were lots of errors concerning the volume of the liquid in the tank at time t. Since the tank starts with 700 gallons, and since the volume increases by 4 gallons per minute (since 5 gallons flow in per minute and since 1 gallon flows out per minute), we get that the volume is $700 + 4t$. If you want to be pedantic about this, you could recognize that the volume is increasing at a rate of 4 gallons per minute, and set up the differential equation $\frac{dV}{dt} = 4$, where V is the volume in the tank (in gallons). Solving this gives $V = 4t + C$ and using the initial condition of 700 gallons, you get $4t + 700$.