Eddie Price **Quiz 6 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 1; Average score: 5

Problem 1 (9 Points). A 1000-gallon tank initially contains 700 gallons of pure, distilled water. Brine containing 4 pounds of salt per gallon ows into the tank at the rate of 5 gallons per minute, and the well-stirred mixture ows out of the tank at the rate of 2 gallons per minute. Find the amount of salt (in pounds) in the tank at the moment the tank becomes full. (Round to 2 decimal places.)

Solution. Let $A(t) = A$ represent the amount of salt in the tank (in lbs) after t minutes.

$$\frac{dA}{dt} = (\text{concentration in})\,(\text{rate in}) - (\text{concentration out})\,(\text{rate out})$$

$$\frac{dA}{dt} = \left(\frac{4 \text{ lbs}}{\text{gal}}\right)\left(\frac{5 \text{ gal}}{\text{min}}\right) - \left(\frac{A \text{ lbs}}{700 + 3t \text{ gal}}\right)\left(\frac{2 \text{ gal}}{\text{min}}\right)$$

(The amount in the tank at time t is A, and the volume is $700 + 3t$ since it starts with 700 gallons and *gains* $5 - 2 = 3$ gallons per minute.)

$$\frac{dA}{dt} = 20 - \frac{2A}{700 + 3t}$$

$$\frac{dA}{dt} + \frac{2}{700 + 3t}A = 20$$

From this, we compute the integrating factor:

$$u(t) = e^{\int \frac{2}{700+3t}\,dt} = e^{\frac{2}{3}\ln|700+3t|} = \left(e^{\ln(700+3t)}\right)^{2/3} = (700 + 3t)^{2/3}$$

(The absolute values can be dropped because our time is always positive). Hence,

$$\int \frac{d}{dt}\left[(700 + 3t)^{2/3} A\right]\,dt = \int 20\,(700 + 3t)^{2/3}\,dt$$

$$(700 + 3t)^{2/3} A = 4\,(700 + 3t)^{5/3} + C$$

$$A(t) = 4\,(700 + 3t) + C\,(700 + 3t)^{-2/3}$$

Since the tank originally contains pure, distilled water, there is *no* salt in the tank at time $t = 0$. So $A(0) = 0$.

$$0 = A(0) = 4 \cdot 700 + C\,(700)^{-2/3}$$

This gives $C = -2800\,(700)^{2/3}$.

$$A(t) = 4\,(700 + 3t) - 2800\,(700)^{2/3}\,(700 + 3t)^{-2/3}$$

The volume is $700 + 3t$. The tank gets full when its volume reaches 1000. This takes place when $t = 100$.

$$A(100) = 4\,(1000) - 2800\,(700)^{2/3}\,(1000)^{-2/3} \approx \boxed{1792.55 \text{ lbs}}$$

Common Mistakes

Many people set up the differential equation incorrectly. In particular, they forgot to include the 2 gallons per minute flowing out.

A lot of people computed the integrating factor incorrectly, particular when integrating $\frac{1}{700+3t}$ or $\frac{2}{700+3t}$. Using a substitution, you will see that you will need a $\frac{1}{3}$ or $\frac{2}{3}$ before the natural log.

A large percentage of the class got the initial condition incorrect when trying to solve for C. The 700 is the initial volume of liquid in the tank, *not the initial amount of salt*. Since the tank originally consisted of pure, distilled water, the initial amount of salt in the tank is 0. So $A(0) = 0$.

A lot of people also incorrectly tried to solve for the final answer, plugging in 1000 for A, which doesn't make sense since A represents the amount of salt in the tank, but 1000 is the volume of the tank when it is full.