**Eddie Price** **Quiz 8 Solutions** **Spring 2018**

**High score: 10; (nonzero) Low score: 1; Average score: 6.61**
Letter grade estimates: **A: 10, B: 9, C+: 8, C: 6-7, C-: 5, D: 3-4, F: 0-2**

Problem 1 (5 Points). Evaluate the integral, and round to 3 decimal places. If the integral diverges, answer "diverges".

$$\int_6^\infty \frac{10}{x\left(4\ln(x)\right)^2}\,dx$$

Solution. By definition, we get

$$\lim_{b\to\infty}\int_6^b \frac{10}{x\left(4\ln(x)\right)^2}\,dx$$

Here, we need to do a $u$-substitution. Let $u = 4\ln(x)$, so $du = \frac{4}{x}\,dx$, giving $\frac{1}{4}\,du = \frac{1}{x}\,dx$.

$$\lim_{b\to\infty}\frac{10}{4}\int_{x=6}^{x=b}\frac{1}{u^2}\,du$$

$$\lim_{b\to\infty}\frac{5}{2}\int_{x=6}^{x=b}u^{-2}\,du$$

$$\lim_{b\to\infty}-\frac{5}{2}u^{-1}|_{x=6}^{x=b}$$

Converting back to $x$, we get

$$\lim_{b\to\infty}-\frac{5}{2}\left(\frac{1}{4\ln(x)}\right)|_6^b = \lim_{b\to\infty}-\frac{5}{2}\left(\frac{1}{4\ln(b)}-\frac{1}{4\ln(6)}\right)$$

We have $\lim_{b\to\infty}\ln(b)=\infty$, so $\lim_{b\to\infty}\frac{1}{4\ln(b)}=0$, so this limit is

$$-\frac{5}{2}\left(0-\frac{1}{4\ln(6)}\right)=\frac{5}{2}\cdot\frac{1}{4\ln(6)}\approx\boxed{0.349}$$

Problem 2 (4 points). Compute the sum of the series.

$$\sum_{n=0}^{\infty}\left(\frac{5\cdot(-1)^{n+1}}{8\cdot 4^n}\right)$$

Solution. $a$ is the first term, which is $\frac{5\cdot(-1)^{0+1}}{8\cdot4^0}=-\frac{5}{8}$, and $r=-\frac{1}{4}$ (which you can find by finding the second term $\frac{5\cdot(-1)^{1+1}}{8\cdot4^1}=\frac{5}{8\cdot4}$ and dividing by $a$: $\frac{5}{8\cdot4}\div-\frac{5}{8}=\frac{5}{8\cdot4}\cdot-\frac{8}{5}=-\frac{1}{4}$). Alternatively, you could write the series as $\sum_{n=0}^{\infty}\frac{-5}{8}\left(\frac{-1}{4}\right)^n$ and use the fact that a geometric series is of the form $\sum_{n=0}^{\infty}ar^n$.

$$\frac{a}{1-r}=\frac{-\frac{5}{8}}{1-\left(-\frac{1}{4}\right)}=\frac{\left(-\frac{5}{8}\right)}{\left(\frac{5}{4}\right)}=-\frac{5}{8}\cdot\frac{4}{5}=\boxed{-\frac{1}{2}}$$

Common Mistakes

For problem 1, a lot of people didn't know how to do the integral correctly. You cannot forget your integration techniques. We saw how to integrate things like this in lesson 3.

For problem 2,some people got $a$ or $r$ wrong.