Eddie Price **Quiz 9 Solutions** **Spring 2018**

High score: 10; (nonzero) Low score: 2; Average score: 7.06
Letter grade estimates: **A: 10, B: 9, C: 8, C-: 6-7, D: 4-5, F: 0-3**

Problem 1 (4 Points). A ball has the property that each time it falls from a height h onto the ground, it will rebound to a height of rh. Find the total distance traveled by a ball with $r = 0.6$ that is dropped from a height of 13 meters.

Solution.

The ball falls 13 m; bounces back $0.6 \cdot 13$ m; falls back down $0.6 \cdot 13$ m; bounces up $0.6 \cdot 0.6 \cdot 13 = 0.6^2 \cdot 13$ m; falls back down $0.6^2 \cdot 13$ m; bounces up $0.6 \cdot 0.6^2 \cdot 13 = 0.6^3 \cdot 13$ m; falls back down $0.6^3 \cdot 13$ m, etc. The total distance it travels is

$$13 + (0.6 \cdot 13 + 0.6 \cdot 13) + \left(0.6^2 \cdot 13 + 0.6^2 \cdot 13\right) + \left(0.6^3 \cdot 13 + 0.6^3 \cdot 13\right) + ...$$
$$13 + 0.6 \cdot 2 \cdot 13 + 0.6^2 \cdot 2 \cdot 13 + 0.6^3 \cdot 2 \cdot 13 + ...$$

This series is not geometric since the first term does not fit the pattern. Ignoring the first term, the rest is a geometric series since every term is 0.6 times the previous term ($r = 0.6$). The first term of the geometric series is $0.6 \cdot 2 \cdot 13$ ($= a$). So the ball travels a distance of

$$13 + \frac{0.6 \cdot 2 \cdot 13}{1 - 0.6} = \boxed{52 \text{ meters}}$$

Problem 2 (5 points). How much money (in dollars) should you invest today at an annual interest rate of 4.2% compounded continuously so that, starting four years from today, you can make annual withdrawals of \$3200 in perpetuity? (Round to the nearest cent.)

Solution. The continuous compound interest formula is $A = Pe^{rt}$ where A is the amount in the account after t years, P is the principal (amount invested), and r is the interest rate. We want to find the principal, so solving for P, we get $P = Ae^{-rt}$. Each year, we want to have \$3200 in the account that we can withdraw, so $A =$ \$3200, and we know that $r = 4.2\% = 0.042$. So $P = 3200e^{-0.042t}$.

To have exactly 3200 in the account four years from now, we need to invest $3200e^{-0.042 \cdot 4}$ right now; to have exactly 3200 in the account five years from now, we need to invest $3200e^{-0.042 \cdot 5}$ right now; to have exactly 3200 in the account six years from now, we need to invest $3200e^{-0.042 \cdot 6}$ right now, etc. So in order to have 3200 in the account four years from now ***and*** five years from now ***and*** six years from now, etc., we need to, right now, invest:

$$3200e^{-0.042 \cdot 4} + 3200e^{-0.042 \cdot 5} + 3200e^{-0.042 \cdot 6} + ...$$

This is a geometric series with $r = e^{-0.042}$ and $a = 3200e^{-0.042 \cdot 4}$.

$$\frac{a}{1 - r} = \frac{3200e^{-0.042 \cdot 4}}{1 - e^{-0.042}} \approx \boxed{\$65,769.95}$$

Common Mistakes

For problem 1, many people forgot the 13 meters the ball originally dropped. Some people remembered it at first, but forgot to add it in for the final answer.

For problem 1, many people forgot the ball bounced up a height and then fell down the same height. This caused them to lose (almost) half of the distance the ball traveled by not incorporating both the bounce up a certain height and the fall of that same height.

For problem 2, many people got a wrong by starting at the wrong place (such as starting one year from now).

For problem 2, many people messed up the compound interest formula used here for principal.

For problem 2, there were many errors in rounding or computation. Make sure you know how to use your calculator correctly and efficiently. Please see the document on the course website about calculators.