Math 266 (Eddie Price) Midterm Exam 2 Study Guide Spring 2019

Exam 1 Material:
You should know all terminology covered on exam 1. You should also be familiar with some basic methods covered on exam 1, since they may be used in some of the methods (e.g., reduction of order) on exam 2. (Mainly should know integrating factor method.)

2nd order linear homogeneous differential equations with constant coefficients:

terminology to know: homogeneous, characteristic polynomial, characteristic equation

You should be able to: find the general solution of a 2nd order linear homogeneous diff eq with constant coefficients, solve IVPs of this sort, analyze solutions to differential equations of this type (understand behavior), find critical values of an initial condition which affect the behavior of solutions, understand what roots the characteristic equation could have to give solutions of certain behavior.

Practice Problems:
- Find the general solution of $y'' - 6y' + 9y = 0$.
- Find the general solution of $y'' - 6y' + 8y = 0$.
- Find the general solution of $y'' - 4y' + 13y = 0$.
- Find the general solution of $y'' + 2y = 0$.
- Solve the IVP $4y'' - y = 0, y(0) = 2, y'(0) = \beta$ and find the value of β so the solution approaches 0 as $t \to \infty$.
- Solve the IVP $9y'' + 12y' + 4y = 0, y(0) = a > 0, y'(0) = 1$ and find the value of a that separates solutions that become negative from those that are always positive.
- Find values of a, b, and c so that the diff eq $ay'' + by' + cy = 0$ has solutions whose behavior exhibits growing oscillations.
- Find values of a, b, and c so that the diff eq $ay'' + by' + cy = 0$ has solutions which tend to 0 as $t \to \infty$ but also have a local maximum or minimum.

Theory of 2nd order linear differential equations:

terminology to know: existence and uniqueness, principle of superposition, fundamental set of solutions, Wronskian

You should be able to: find the largest open intervals on which a unique solution to an IVP exists, apply the principle of superposition, know what a fundamental set is, determine whether a set of solutions is a fundamental set, understand the connections between fundamental sets and general solutions, compute the Wronskian.

Practice Problems:
- Find the largest open interval on which the IVP is guaranteed to have a unique solution: $(x-2)y'' + y' + (x-2)(\tan x)y = 0$, $y(3) = 1$, $y'(3) = 2$
- If $e^{t/2}$ and $e^{-t/2}$ are solutions to a differential equation, prove that $\sinh(t/2)$ is a solution as well.
- Suppose e^t and te^t are solutions to a second order linear diff eq. Show that $c_1e^t + c_2te^t$ includes all solutions to the differential equation.

Reduction of Order:

terminology to know: method of reduction of order

You should be able to: use the method of reduction of order to find the general solution to a 2nd order linear diff eq (with nonconstant coefficients) given one solution

Practice Problems:
- Find the general solution of $t^2y'' + 3ty' + y = 0, t > 0$ given that $y_1 = t^{-1}$ is a solution.

Nonhomogeneous diff eqs:

terminology to know: nonhomogeneous, nonhomogeneous part, complementary solution, particular solution, undetermined coefficients, variation of parameters

You should be able to: determine the form of a particular solution using the method of undetermined coefficients, find a particular solution/general solution using the method of undetermined coefficients, find a particular solution/general solution using the method of variation of parameters

Practice Problems:
- Find the form of a particular solution for the diff eq $y'' - 2y' - 3y = e^{3t} + 4t^3e^{-t} + 5\cos(2t)$
- Find the form of a particular solution for the diff eq $y'' - 2y' = 8t^3 + t + e^t \sin(t) + te^{-2t}$
- Find the general solution to the diff eq $y'' + 9y = \cos(2t)$.
- Find the general solution to $ty'' - (1 + t)y' + y = t^2e^{2t}$, $t > 0$, given that $y_1 = 1 + t$ and $y_2 = e^t$ form a fundamental set of solutions to the corresponding homogeneous diff eq.

Mass-Spring Systems and Vibrations:

terminology to know: mass-spring system, damping coefficient, spring coefficient, Newtons, dynes, undamped vibrations, damped vibrations, free vibrations, forced vibrations, amplitude, period, natural frequency, phase shift, underdamped, critically damped, overdamped, quasi-frequency, quasi-period, forcing function, transient solution, steady state solution, beats/amplitude modulation, resonance

You should be able to: set up an IVP representing the motion of a mass-spring system whether it is undamped or damped and whether it is free or forced, keep units consistent, solve basic mass-spring system IVPs, convert $A\cos(\alpha t) + B\sin(\alpha t)$ to $R\cos(\omega_0 t - \delta)$ form, find the amplitude, period, natural frequency, and phase shift of an undamped free system, determine whether a free damped system is critically damped, overdamped, or underdamped, determine the value of the damping coefficient for which the system is critically damped, find the quasi-period and quasi-frequency of an underdamped free system, determine the transient and steady state solutions for a damped forced system, determine the frequency of the forcing function required for an undamped forced system to experience resonance, describe the conditions necessary to get beats.

Practice Problems:
- A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external foce of $10\sin(t/2)$ N (Newtons) and moves in a medium that imparts a viscuous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s, formulate the initial value problem describing the motion of the mass. (Specificy the units of u.)
- Convert $3\cos 2t + 4\sin 2t$ into $R\cos(\omega_0 t - \delta)$ form.
- A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in and then released, and if there is no damping, determine the natural frequency, period, phase, and amplitude of the resulting motion.
- A mass of 20 g stretches a spring 5 cm. Suppose that the mass is also attaches to a viscous damper with a damping constant of 400 dyn·s/cm. If the mass is pulled down an additional 2 cm and then released, determine the quasi-frequency and quasi-period of the resulting motion.
- A mass weighing 8 lb stretches a spring 1.5 in. The mass is also attached to a damper with coefficient γ. Determine the value of γ for which the system is critically damped; be sure to give the units for γ. State all possible values (in interval form) of γ for which the system is underdamped; do the same for when the system is overdamped.
- Suppose a mass-spring system is represented by $u'' + u' + 1.25u = 3\cos t$ subject to the initial condition $u(0) = 2, u'(0) = 3$. Find the steady state and transient solutions. Is the motion asymptotic to the steady state or to the transient solution?
- Suppose a mass-spring system is represented by $u'' + u = 0.5\cos 0.8t, u(0) = 0, u'(0) = 0$. Does the system experience beats or resonance?
- Suppose a mass-spring system is represented by $u'' + u = 3\cos\omega t$. Find the value of ω for which the system experiences resonance.

<u>Higher Order Linear Diff Eqs</u>:

You should be able to: find the largest open interval on which an nth order linear IVP with constant coefficients has a unique solution, solve an nth order homogeneous linear diff eq with constant coefficients

Practice Problems:
- Determine the largest intervals on which solutions are sure to exist: $t(t-1)y^{(4)} + e^t y'' + 4t^2 y = 0$.
- Find the general solution: $2y''' - 4y'' - 2y' + 4y = 0$.
- Find the general solution: $y^{(6)} - y'' = 0$.
- Find the general solution: $y^{(4)} + 2y'' + y = 0$.

<u>Problems you can also do from the course website's study guides</u>
Study Guide 1: 25, 26, 27
Study Guide 2: 1-15, 18, 21