Problem 1 (20 Points). Evaluate \( \iint_S x^2yz \, dS \), where \( S \) is the part of the plane \( z = 1 + 2x + 3y \) that lies above the rectangle \([0, 3] \times [0, 2]\).

Solution.
\( S \) can be parameterized as \( \vec{r}(x, y) = (x, y, 1 + 2x + 3y), 0 \leq x \leq 3, 0 \leq y \leq 2 \).

To find the surface area differential \( dS \), you could either do \(|\vec{r}_x \times \vec{r}_y| \, dA\) or recognize that since we have \( z \) as a function of \( x \) and \( y \), we could use \( \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \, dA \). In either case, we get \( dS = \sqrt{14} \, dA \).

By the parameterization of \( S \), we get \( x^2yz = x^2y(1 + 2x + 3y) = x^2y + 2x^3y + 3x^2y^2 \). Thus, we get

\[
\iint_S x^2yz \, dS = \iint_D (x^2y + 2x^3y + 3x^2y^2) \sqrt{14} \, dA
\]

\[
= \sqrt{14} \int_0^2 \int_0^3 (x^2y + 2x^3y + 3x^2y^2) \, dx \, dy
\]

\[
= \sqrt{14} \int_0^2 \left( \frac{1}{3}x^3y + \frac{1}{2}x^4y + x^3y^2 \right) \bigg|_{x=0}^{x=3} \, dy
\]

\[
= \sqrt{14} \int_0^2 \left( 9y + \frac{81}{2}y + 27y^2 \right) \, dy
\]

\[
= \sqrt{14} \left( \frac{9}{2}y^2 + \frac{81}{4}y^2 + 9y^3 \right) \bigg|_0^2
\]

\[
= \sqrt{14} (9 \cdot 2 + 81 + 9 \cdot 8) = \sqrt{14} (18 + 81 + 72) = 171\sqrt{14}
\]