Eddie Price **Quiz 10 Solutions** **Summer 2018**

High score: 20; (nonzero) Low score: 12; Average score: 18.07

Problem 1 (20 Points). Evaluate $\iint_S x^2yz\,dS$, where S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $[0, 3] \times [0, 2]$.

Solution.
S can be parameterized as $\vec{r}(x, y) = \langle x, y, 1 + 2x + 3y\rangle, 0 \le x \le 3, 0 \le y \le 2$.

To find the surface area differential dS, you could either do $|\vec{r}_x \times \vec{r}_y|\ dA$ or recognize that since we have z as a function of x and y, we could use $\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\ dA$. In either case, we get $dS = \sqrt{14}\,dA$.

By the parameterization of S, we get $x^2yz = x^2y\,(1 + 2x + 3y) = x^2y + 2x^3y + 3x^2y^2$. Thus, we get

$$\iint_S x^2yz\,dS = \iint_D \left(x^2y + 2x^3y + 3x^2y^2\right)\sqrt{14}\,dA$$

$$= \sqrt{14}\int_0^2\int_0^3 \left(x^2y + 2x^3y + 3x^2y^2\right)\,dx\,dy$$

$$= \sqrt{14}\int_0^2 \left(\frac{1}{3}x^3y + \frac{1}{2}x^4y + x^3y^2\right)\Bigg|_{x=0}^{x=3}\,dy$$

$$= \sqrt{14}\int_0^2 \left(9y + \frac{81}{2}y + 27y^2\right)\,dy$$

$$= \sqrt{14}\left(\frac{9}{2}y^2 + \frac{81}{4}y^2 + 9y^3\right)\Bigg|_0^2$$

$$= \sqrt{14}\,(9\cdot 2 + 81 + 9\cdot 8) = \sqrt{14}\,(18 + 81 + 72) = \boxed{171\sqrt{14}}$$