**Eddie Price** **Quiz 1 Solutions** **Summer 2018**

**High score: 20; (nonzero) Low score: 7; Average score: 16.52**

Problem 1 (8 Points). Let $\mathbf{u} = \langle 1, 0, 5\rangle$ and $\mathbf{v} = \langle -2, 6, 1\rangle$. Find
(a) $2\mathbf{u}$, (b) $\mathbf{u} - \mathbf{v}$, (c) $\mathbf{u} \cdot \mathbf{v}$, and (d) $\mathbf{u} \times \mathbf{v}$

Solution.

(a): $2\mathbf{u} = 2\langle 1,0,5\rangle = \langle 2\cdot 1, 2\cdot 0, 2\cdot 5\rangle = \boxed{\langle 2, 0, 10\rangle}$

(b): $\mathbf{u} - \mathbf{v} = \langle 1,0,5\rangle - \langle -2,6,1\rangle = \langle 1-(-2), 0-6, 5-1\rangle = \boxed{\langle 3, -6, 4\rangle}$

(c): $\mathbf{u}\cdot\mathbf{v} = (1)(-2) + (0)(6) + (5)(1) = -2 + 0 + 5 = \boxed{3}$

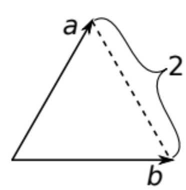
(d): $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 5 \\ -2 & 6 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 5 \\ 6 & 1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 5 \\ -2 & 1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 0 \\ -2 & 6 \end{vmatrix}\mathbf{k}$

$= (0-30)\mathbf{i} - (1-(-10))\mathbf{j} + (6-0)\mathbf{k} = \boxed{\langle -30, -11, 6\rangle}$

Problem 2 (6 points). Find a value for $t$ such that the vectors $18\mathbf{i} + t\mathbf{j}$ and $2\mathbf{i} - 4t\mathbf{j}$ are perpendicular.

Solution. Two vectors are perpendicular if and only if their dot product is 0. As such, we seek to find a value of $t$ so that $(18)(2) + (t)(-4t) = 0$, or equivalently, when $36 - 4t^2 = 0$. This is equivalent to $t^2 = 9$. Hence, the vectors will be perpendicular if $\boxed{t = 3 \text{ or } t = -3}$.

Problem 3 (6 points). The vectors $\mathbf{a}$ and $\mathbf{b}$ form two sides of the equilateral triangle shown below whose sides are of length 2. Find $|\mathbf{a}\times\mathbf{b}|$. Should $\mathbf{a}\times\mathbf{b}$ point into or out of the page?




Solution. Recall that $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin\theta$, where $\theta$ is the angle between the two vectors. In the case of an equilateral triangle, all angles are congruent, and hence, must be 60°.

$$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin\theta = 2\cdot 2\cdot\sin(60^\circ) = 2\cdot 2\cdot\frac{\sqrt{3}}{2} = \boxed{2\sqrt{3}}$$

If you take your right hand, point your fingers in the direction of $\mathbf{a}$, and curl them toward $\mathbf{b}$, then your thumb points into the page. Hence, according to the right hand rule, $\mathbf{a}\times\mathbf{b}$ points $\boxed{\text{into the page}}$.

(If you forgot the formula for the magnitude of the cross product, you could also find it using the fact that the area of the triangle is equal to $\frac{1}{2}|\mathbf{a}\times\mathbf{b}|$, and using geometry to find the area of the triangle.)