Problem 1 (20 Points). Let \( r(t) = te^t i - 2j + \sin(t) k \).

(a) Find \( r'(t) \).
(b) Find a vector equation for the line \( u(t) \) tangent to the curve at the point where \( t = 0 \).

Solution.

(a): We can differentiate a vector function component-wise, so we get

\[
 r'(t) = (te^t + e^t) i - 0j + \cos(t) k = (te^t + e^t) i + \cos(t) k
\]

(b): To get a vector equation for a line, we need a point on the line and a vector that is in the same direction as the line. The point is the tip of \( r(0) = 0i - 2j + 0k \), so \((0, -2, 0)\). Since the line is tangent to the curve at \( t = 0 \), it should be in the same direction as the tangent vector to the curve at \( t = 0 \). In other words, a vector pointing in the same direction as the line is \( r'(0) = i + k \).

The general formula for a vector equation of the line is \( OP + tv \), where \( O \) is the origin, \( P \) is the point, and \( \vec{v} \) is the vector in the direction of the line. Hence,

\[
 u(t) = (-2j) + t (i + k) = ti - 2j + tk
\]