Eddie Price **Quiz 2 Solutions** **Summer 2018**

High score: 20; (nonzero) Low score: 10; Average score: 13.59

Problem 1 (20 Points). Let $\mathbf{r}(t) = te^t\mathbf{i} - 2\mathbf{j} + \sin(t)\,\mathbf{k}$.
(a) Find $\mathbf{r}'(t)$.
(b) Find a vector equation for the line $\mathbf{u}(t)$ tangent to the curve at the point where $t = 0$.

Solution.

(a): We can differentiate a vector function component-wise, so we get

$$\mathbf{r}'(t) = \left(te^t + e^t\right)\mathbf{i} - 0\mathbf{j} + \cos(t)\,\mathbf{k} = \boxed{\left(te^t + e^t\right)\mathbf{i} + \cos(t)\,\mathbf{k}}$$

(b): To get a vector equation for a line, we need to a point on the line and a vector that is in the same direction as the line. The point is the tip of $\mathbf{r}(0) = 0\mathbf{i} - 2\mathbf{j} + 0\mathbf{k}$, so $(0, -2, 0)$.

Since the line is tangent to the curve at $t = 0$, it should be in the same direction as the tangent vector to the curve at $t = 0$. In other words, a vector pointing in the same direction as the line is $\mathbf{r}'(0) = \mathbf{i} + \mathbf{k}$.

The general formula for a vector equation of the line is $\vec{OP} + t\mathbf{v}$, where O is the origin, P is the point, and $\vec{v}$ is the vector in the direction of the line. Hence,

$$\mathbf{u}(t) = (-2\mathbf{j}) + t\,(\mathbf{i} + \mathbf{k}) = \boxed{t\mathbf{i} - 2\mathbf{j} + t\mathbf{k}}$$