Eddie Price **Quiz 3 Solutions** **Summer 2018**

High score: 20; (nonzero) Low score: 7; Average score: 15.06

Problem 1 (20 Points). Find the length of the curve $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leq t \leq 1$.

Solution.

Recall that $L = \int_0^1 |\mathbf{r}'(t)|\ dt$, so we first find $\mathbf{r}'(t)$:

$$\mathbf{r}'(t) = 2t\mathbf{j} + 3t^2\mathbf{k}$$

$$|\mathbf{r}'(t)| = \sqrt{(2t)^2 + (3t^2)^2} = \sqrt{4t^2 + 9t^4} = \sqrt{t^2(4 + 9t^2)} = |t|\sqrt{4 + 9t^2} = t\sqrt{4 + 9t^2}$$

($|t| = t$ since $0 \leq t \leq 1$)

$$L = \int_0^1 t\sqrt{4 + 9t^2}\, dt$$

Make the following u-substitution: $u = 4 + 9t^2$, $du = 18t\, dt$. Also, $u(0) = 4$, $u(1) = 13$.

$$\frac{1}{18}\int_4^{13} u^{1/2}\, du$$

$$= \frac{1}{18} \cdot \frac{2}{3} u^{3/2} \Big|_4^{13} = \frac{1}{27}\left(13^{3/2} - 4^{3/2}\right) = \boxed{\frac{1}{27}\left(13^{3/2} - 8\right)}$$