

**High score: 20; (nonzero) Low score: 2; Average score: 12.55**

Problem 1 (20 Points). Set up **but do not evaluate** an iterated integral to compute  $\iint_D y^2 dA$  where  $D$  is the triangular region with vertices  $(0, 1)$ ,  $(1, 2)$ ,  $(4, 1)$ .

Solution.

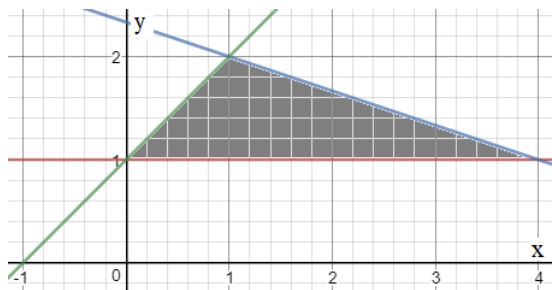


Figure 1: Image made in Desmos and Paint

Notice that we would need to split this up into two double integrals if we did  $dy dx$ , so we will do  $dx dy$ .

The blue line contains the points  $(1, 2)$  and  $(4, 1)$ , so it has a slope  $\frac{1-2}{4-1} = -\frac{1}{3}$ . Hence, the line has equation  $y - 2 = -\frac{1}{3}(x - 1)$ . Solving for  $x$ , we get  $-3y + 6 = x - 1$  or  $x = -3y + 7$ .

The green line contains the points  $(0, 1)$  and  $(1, 2)$ , so it has a slope  $\frac{2-1}{1-0} = 1$ . Hence, the line has equation  $y - 1 = x$ . So  $x = y - 1$ .

We see that  $D$  is bounded on the left by curve  $x = y - 1$  and on the right by curve  $x = -3y + 7$ . The minimum  $y$ -value in  $D$  is 1, and the maximum  $y$ -value in  $D$  is 2. Hence, we get the iterated integral

$$\int_1^2 \int_{y-1}^{-3y+7} y^2 dx dy$$