Problem 1 (20 Points). Set up but do not evaluate an iterated integral to compute \( \int_D y^2 \, dA \) where \( D \) is the triangular region with vertices \((0, 1), (1, 2), (4, 1)\).

Solution.

![Figure 1: Image made in Desmos and Paint](image)

Notice that we would need to split this up into two double integrals if we did \( dy \, dx \), so we will do \( dx \, dy \).

The blue line contains the points \((1, 2)\) and \((4, 1)\), so it has a slope \( \frac{2-1}{1-0} = 1 \). Hence, the line has equation \( y - 2 = \frac{1}{3} (x - 1) \). Solving for \( x \), we get \(-3y + 6 = x - 1\) or \( x = -3y + 7\).

The green line contains the points \((0, 1)\) and \((1, 2)\), so it has a slope \( \frac{2-1}{1-0} = 1 \). Hence, the line has equation \( y - 1 = x \). So \( x = y - 1\).

We see that \( D \) is bounded on the left by curve \( x = y - 1 \) and on the right by curve \( x = -3y + 7\). The minimum \( y \)-value in \( D \) is 1, and the maximum \( y \)-value in \( D \) is 2. Hence, we get the iterated integral

\[
\int_{1}^{2} \int_{y-1}^{-3y+7} y^2 \, dx \, dy
\]