Eddie Price **Quiz 6 Solutions** **Summer 2018**

High score: 20; (nonzero) Low score: 5; Average score: 18.77

Problem 1 (20 Points). Evaluate $\iiint_R (x^2 + y^2)\ dV$, where R is the region between the spheres of radius 1 and radius 2 centered at the origin.

$$Hint: \int_0^\pi \sin^3 u\, du = \frac{4}{3}$$

Solution. In spherical coordinates, a sphere of radius a centered at the origin has the equation $\rho = a$. There are no restrictions on θ or on ϕ, so they can take on their full ranges: $0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq \pi$.

$$\int_0^\pi \int_0^{2\pi} \int_1^2 \left(\rho^2 \cos^2\theta \sin^2\phi + \rho^2 \sin^2\theta \sin^2\phi\right) \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

Notice that the integrand contains $\left(\cos^2\theta + \sin^2\theta\right) \rho^2 \sin^2\phi \cdot \rho^2 \sin\phi$, so we get

$$\int_0^\pi \int_0^{2\pi} \int_1^2 \rho^4 \sin^3\phi\, d\rho\, d\theta\, d\phi$$

$$= \int_0^\pi \int_0^{2\pi} \frac{1}{5}\rho^5 \sin^3\phi \Big|_{\rho=1}^{\rho=2} d\theta\, d\phi$$

$$= \int_0^\pi \int_0^{2\pi} \frac{32-1}{5} \sin^3\phi\, d\theta\, d\phi$$

$$= \frac{31}{5} \int_0^\pi \sin^3\phi \int_0^{2\pi} d\theta\, d\phi$$

$$= \frac{31}{5} \cdot 2\pi \int_0^\pi \sin^3\phi\, d\phi$$

By the hint, we get

$$= \frac{31}{5} \cdot 2\pi \cdot \frac{4}{3} = \boxed{\frac{248}{15}\pi}$$