**Eddie Price** **Quiz 8 Solutions** **Summer 2018**

**High score: 20; (nonzero) Low score: 15; Average score: 17.34**

Problem 1 (20 Points). Use Green's Theorem to evaluate $\int_C y^3\,dx - x^3\,dy$, where $C$ is the positively-oriented circle $x^2 + y^2 = 4$.

Solution.

Let $D$ be the region enclosed by $C$. Since $C$ is a positively-oriented closed curve, by Green's Theorem, we have

$$\oint_C y^3\,dx - x^3\,dy = \iint_D \left( \frac{\partial}{\partial x}\left[-x^3\right] - \frac{\partial}{\partial y}\left[y^3\right] \right)\,dA = \iint_D \left(-3x^2 - 3y^2\right)\,dA$$

Since the region $D$ is the interior of the circle $x^2 + y^2 = 4$, this double integral is best done in polar coordinates:

$$\int_0^{2\pi} \int_0^2 -3r^2 \cdot r\,dr\,d\theta$$

$$= -3 \int_0^{2\pi} \frac{1}{4} r^4 \Big|_{r=0}^{r=2}\,d\theta$$

$$= -3 \cdot 4 \int_0^{2\pi} d\theta$$

$$= -12 \cdot 2\pi = \boxed{-24\pi}$$