

**High score: 20; (nonzero) Low score: 12; Average score: 19.31**

Problem 1 (20 Points). Find a parametric representation for the part of the surface  $x^2 + y^2 = 4z^2$  that lies above the  $xy$ -plane.

Solution.

There are many valid solutions one could give. For example, one could notice that in order to be above the  $xy$ -plane, we need  $z > 0$ . As such, we could solve the above equation for  $z$ :

$$z = \sqrt{\frac{x^2}{4} + \frac{y^2}{4}} = \frac{\sqrt{x^2 + y^2}}{2}$$

You could then have the following parameterization:

$$x = x, \quad y = y, \quad z = \frac{\sqrt{x^2 + y^2}}{2}, \quad 0 < x^2 + y^2$$

Alternatively, you could try viewing the surface in cylindrical coordinates. In this case,  $x = r \cos \theta$ ,  $y = r \sin \theta$ , and  $r^2 = 4z^2$ . Solving for  $z$  and noticing that we want  $z > 0$ , we get the parameterization:

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = \frac{r}{2}, \quad r > 0, \quad 0 \leq \theta < 2\pi$$