Pricing an Annuity

Central Indiana Life Insurance Company 's customers can use a portion of the funds accumulated in their 401(k) retirement plan to buy an annuity which pays $30,000 a year until death.

Part 1: When Martin Dempster retired at age 61 on January 1, 2001, he opted to purchase the above mentioned annuity. His first payment is made immediately. The actuaries at CILIC predicted that interest rates would average 5% per year over the life of the annuity. You have been assigned to compute how much CILIC should charge Mr. Dempster for the annuity. (For sake of simplicity, we will assume that the payments are made once a year on January 1, although monthly payments would be more typical.)

Discussion: Suppose, for example, we knew that Martin would die between the 5th and 6th payments. Then, CILIC would make the following payments:

Year	1	2	3	4	5
Payment	30,000	30,000	30,000	30,000	30,000

At 5% interest, the present value of these payments is

$30000+30000(1.05)^{-1}+30000(1.05)^{-2}+30000(1.05)^{-3}+30000(1.05)^{-4}=106,378.52$

which is the least we would charge for the annuity. (We would certainly charge more to cover administrative costs and to allow for a modest profit.)

However, we have no way of knowing how long Dempster will live. We solve this problem using what is referred to as a *mortality table*. This table lists, for each age, the fraction of people that age who typically will die before their next birthday. The “Die” column on page 3 was copied from a mortality table for males.

Imagine, then, that we have 1000 customers, all of whom retire on January 1 at age 61. The entry in the Die column corresponding to age 61 is 0.01705. Hence, of each 1000, 61 year olds, on the average, 17.05 will die before reaching age 62. The fraction surviving is

$$1-0.01705=.98295$$

so 982.95 will survive to age 62.

According to the Die column, the fraction of 62 year olds dying before age 63 is 0.0187 so the fraction surviving is

$$1-0.0187=.9813$$

and we expect

.9813*982.95=964.60

still alive in 2003. Continuing in this way, we can compute, for each year after 2001, how many of our original 1000 we expect to be still alive. We pay each of these survivors his $30,000 and take the present value of the total. This is our cost for this year. We sum all of these costs and divide by the number of annuities sold (1,000) to get the cost of one annuity. Our results are listed in the spreadsheet on p. 3 of this project. **Your first assignment is to reproduce this spread sheet out to age 119.**

Specifically, after filling out the first 15 rows as indicated, download the mortality table MortTab.xls from the class website (www.math.purdue.edu/~rcp/MathStat301.html) and copy the portion of the mortality table beginning with row 61 onto the third column of your spread sheet, beginning with cell C17. (This will be demonstrated in class.) Then compute the other columns as follows

1.) The fraction surviving column (D) is (1-the "Die" column (C)).

2.) The E17 entry of the People column is 1000 and each subsequent entry is computed by multiplying the People in the preceding year by the fraction surviving in the preceding year.

3.) The amount paid out (paid/10000) is the number of people still alive times 3. (We divide by 10000 to make the numbers fit better in the column.)

4.) The principal value (PV) is the amount paid out times $(1+i)^{-n}$ where i is the interest rate (in this case 5%) and n is the year number.

5.) H17 is the same as H16. Subsequent values of column H are obtained by adding the previous value to the current value of the PV column.

Part 2: Price an annuity where the data is as given next to your name on the chart on the last page of this project. You will need to use the column of the mortality table corresponding to the sex of your annuitant.

e-mail the completed project as an attachment toJenni at **jenni_pi@hotmail.com**. ***Lab projects must be mailed before midnight of the Monday following the lab.***

Class: Math/Stat 170

Lab: 2

Date: 9/1/02

Central Indiana Life Insurance Co.

Yearly Payment:	$30,000
Annuitant Age:	61
Assumed Rate:	5%
Population Size:	1000
Sex:	Male
Price of Annuity	#REF!

Year	Age	Die	Survive	People	Paid/10000	PV/10000	Total PV/10000 to date
0	61	0.0170	0.983	1000	$ 3,000	$ 3,000	$ 3,000.00
1	62	0.0187	0.9813	983	$ 2,949	$ 2,808	$ 5,808.43
2	63	0.0205	0.9795	964.6	$ 2,894	$ 2,625	$ 8,433.20
3	64	0.0225	0.9775	944.8	$ 2,834	$ 2,449	$ 10,881.73
4	65	0.0248	0.9752	923.5	$ 2,771	$ 2,279	$ 13,161.11
5	66	0.0272	0.9728	900.6	$ 2,702	$ 2,117	$ 15,278.13
6	67	0.0297	0.9703	876.1	$ 2,628	$ 1,961	$ 17,239.48
7	68	0.0322	0.9678	850.1	$ 2,550	$ 1,812	$ 19,051.97
8	69	0.0348	0.9652	822.7	$ 2,468	$ 1,671	$ 20,722.57
9	70	0.0376	0.9624	794.1	$ 2,382	$ 1,536	$ 22,258.27
10	71	0.0408	0.9592	764.2	$ 2,293	$ 1,408	$ 23,665.80
11	72	0.0443	0.9557	733.1	$ 2,199	$ 1,286	$ 24,951.62
12	73	0.0483	0.9517	700.6	$ 2,102	$ 1,170	$ 26,121.90
13	74	0.0527	0.9473	666.7	$ 2,000	$ 1,061	$ 27,182.61
14	75	0.0575	0.9425	631.6	$ 1,895	$ 957	$ 28,139.54
15	76	0.0627	0.9373	595.2	$ 1,786	$ 859	$ 28,998.48
16	77	0.0683	0.9317	557.9	$ 1,674	$ 767	$ 29,765.19
17	78	0.0741	0.9259	519.8	$ 1,559	$ 680	$ 30,445.55

	Last	First	Mid	Rate	Age	Sex
1	ADAMS	DUSTIN	K	0.055	40	M
2	ASCHLIMAN	LANCE	K	0.06	41	M
3	AUSTIN	CHRISTOP	D	0.065	42	M
4	BAU	HSI-HSUA		0.07	43	M
5	BEDEL	RYAN	M	0.075	44	M
6	BOTTERON	JOSEPH	M	0.055	45	M
7	BUDIMAN	LIA	I	0.06	46	M
8	CANTRELL	SETH	O	0.065	47	M
9	GERBER	KARYN	S	0.07	48	M
10	GUYMON	MEGHAN	E	0.075	49	M
11	HICKS	JENNIFER	V	0.055	50	M
12	HOLLONQUEST	YANCY	L	0.06	51	M
13	HOSTETTER	ANNE	M	0.065	52	M
14	JOHNSON	ADAM	P	0.07	53	M
15	JOHNSON	ELIZABET	A	0.075	54	M
16	JUNG	JIYOUNG		0.055	55	F
17	KEEGAN	CHRISTIN	D	0.06	56	F
18	KETTELL	JESSICA	J	0.065	57	F
19	KING	WILLIAM	D	0.07	58	F
20	MASLAN	ZULFIQAR		0.075	59	F
21	MAVES	JESSICA	M	0.055	63	F
22	MOSLEY	ERIK	A	0.06	64	F
23	OSHAUGHNESSE	KARA	R	0.065	65	F
24	PAVLECICH	JOSEPH	J	0.07	66	F
25	PUND	BROOKE	L	0.075	67	F
26	QUERY	KEVIN	C	0.055	68	F
27	SCOTT	JASON	H	0.06	69	F
28	SERDAR	NICHOLAS	D	0.065	70	F
29	SEWANI	FAIZAN	K	0.07	71	F
30	SHAH	VIKAS	P	0.075	72	F
31	STACKHOUSE	LANAYA	S	0.55	73	F
32	TANG	HSIAO-FA		0.065	74	M
33	THOMSON	MEGAN	A	0.07	75	M
34	WATKINS	DARREN	L	0.075	76	M
35	WILSON	BRANDON	J	0.055	77	M
36	ZHANG	XIAOLI		0.06	78	M
37	?	?	?	0.065	79	M
38	?	?	?	0.07	80	M
39	?	?	?	0.075	81	M
40	?	?	?	0.55	82	M