

$$\begin{aligned} &> \text{FV} := (i, PV, n, Y) \rightarrow (1+i)^n PV - ((1+i)^n - 1) Y / i; \\ &FV := (i, PV, n, Y) \rightarrow (1+i)^n PV - \frac{((1+i)^n - 1) Y}{i} \end{aligned} \quad (1)$$

$$\begin{aligned} &> \text{solve}(\text{FV}(.048, PV, 3, 0) = 120000, PV); \\ &1.042551213 \cdot 10^5 \end{aligned} \quad (2)$$

#1. Jason invested \$1300 in a mutual fund at the beginning of 1981 and it was worth \$3750 at the beginning of 1996. Find the annual effective interest rate that would have produced the same growth over the 15 year period.

$$\begin{aligned} &> (3750.0/1300)^{(1/15)} - 1; \\ &0.073179894 \end{aligned} \quad (3)$$

#2. Morgan has \$12,000 to invest for five years. North Bank offers a five year CD (Certificate of Deposit) that pays 7% compounded monthly. South Bank offers two year CDs that pay 6% compounded daily and three year CDs that pay 8% compounded quarterly. If he assumes the interest rates will be the same in two years as now, how much more or less would he earn at South Bank by investing in a two year CD then reinvesting the proceeds at maturity in a three year CD compared with investing in a five year CD at North Bank?

North Bank:

$$\begin{aligned} &> (1+.07/12)^{(5*12)} * 12000; \\ &17011.50277 \end{aligned} \quad (4)$$

South Bank:

$$\begin{aligned} &> (1+.08/4)^{(3*4)} * (1+.06/365)^{(2*365)} * 12000; \\ &17159.09983 \end{aligned} \quad (5)$$

Difference:

$$\begin{aligned} &> 17159.09983 - 17011.50277 \\ &147.59706 \end{aligned} \quad (6)$$

#3. In the preceding problem, what is the annual effective yield obtained by investing in a two year CD and then reinvesting the proceeds at maturity in a three year CD at South Bank-i.e. what annual effective interest rate would have produced the same growth over the 5 year period?

$$\begin{aligned} &> ((1+.08/4)^{(3*4)} * (1+.06/365)^{(2*365)})^{(1/5)} - 1; \\ &0.074144356 \end{aligned} \quad (7)$$

#4. Alicia has been depositing \$225 at the end of each month since September 30, 1992. The account pays 6% compounded monthly and she has made no other deposits or withdrawals. How much will she have in the account on December 31, 2002?

$$\begin{aligned} &> i := .06/12; \\ &i := 0.005000000000 \end{aligned} \quad (8)$$

$$\begin{aligned} &> ((1+i)^{(4+12*10)} - 1) * 225 / i; \\ &38522.63200 \end{aligned} \quad (9)$$

#5. Bob R. is offering "low, low" 3.9% financing for 60 month new car loans. If you need to borrow \$18,000 to buy the car of your dreams, what will your monthly payments be?

$$\begin{aligned} &> i := .039/12; \\ &i := 0.003250000000 \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{solve}(0 = (1+i)^{60} * 18000 - ((1+i)^{60} - 1) * Y / i, Y); \\ &330.6857208 \end{aligned} \quad (11)$$

#6. Roberta Johnson needs to borrow \$175,000 to buy a new home. Purdue Saving's bank offered her a 25 year mortgage with monthly payments of \$1432.82 to finance her purchase. The nominal annual interest rate, compounded monthly, is (a) 8.7%, (b) 8.71%, or (c) 8.72%? (Choose the correct response and justify your answer.)

We try each value of i:

$$\begin{aligned} &> i := .0872/12; \\ &i := 0.007266666667 \end{aligned} \quad (12)$$

$$\begin{aligned} &> (1+i)^{(25*12)} * 175000 - ((1+i)^{(25*12)} - 1) * 1432.82 / i; \\ &2532.294 \end{aligned} \quad (13)$$

$$\begin{aligned} &> i := .0871/12; \\ &i := 0.007258333333 \end{aligned} \quad (14)$$

$$\begin{aligned} &> (1+i)^{(25*12)} * 175000 - ((1+i)^{(25*12)} - 1) * 1432.82 / i; \\ &1259.238 \end{aligned} \quad (15)$$

$$\begin{aligned} &> i := .087/12; \\ &i := 0.007250000000 \end{aligned} \quad (16)$$

$$> (1+i)^{(25*12)}*175000-((1+i)^{(25*12)}-1)*1432.82/i;$$

$$-9.266 \quad (17)$$

Since this is closest to 0, (a) is the correct answer.

#7. Starting January 31, 1987, Morris deposited \$1000 per month into an account that pays 7.5% compounded monthly, with the final deposit to occur on December 31, 1996. Starting January 31, 1997, and the last day of each month thereafter, he will withdraw \$1500. On what date will his last withdrawal be? What is the remaining balance after the final withdrawal?

Balance on Dec. 31, 1996

$$> i:=.075/12;$$

$$i:=0.006250000000 \quad (18)$$

$$> ((1+i)^{(10*12)}-1)*1000/i;$$

$$1.779303419 \cdot 10^5 \quad (19)$$

Find number of months:

$$> \text{solve}((1+i)^n*177930.3419-((1+i)^n-1)*1500/i=0,n);$$

$$217.0565528 \quad (20)$$

Hence, we make 217 withdrawals. Find final balance:

$$> (1+i)^{217}*177930.3419-((1+i)^{217}-1)*1500/i;$$

$$84.5504 \quad (21)$$

Find year of last withdrawal:

$$> 217.0565528/12;$$

$$18.08804607 \quad (22)$$

Find month of last withdrawal:

$$> 217.0565528-18*12;$$

$$1.0565528 \quad (23)$$

Hence, the last withdrawal will be on Jan. 31, 2015.

#8. On Jan. 1 you won a contest that pays \$100 dollars at the end of each month for the next 5 years together with an additional payment of \$1,000 at the end of the last month. Two years later, on Jan. 1, I offer to buy the rights to the remaining payments. What is the least amount you should make me pay, given that you can invest money in the bank at 3% interest, compounded monthly?

If you had kept the contest money for the last 3 years, you would have received:

$$> i:=.03/12;$$

$$i:=0.002500000000 \quad (24)$$

$$> 1000+((1+i)^{(12*3)}-1)*100/i;$$

$$4762.056040 \quad (25)$$

You should charge me at least the present value:

$$> (1+i)^{(-12*3)}*4762.056040;$$

$$4352.680356 \quad (26)$$

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9. The following chart shows the deposits and withdrawals in an account that earns $i\%$ compound interest per year. The balance on Jan. 1 was \$1000 and the Dec. 31 balance was \$1500. Approximate i .

March 1	July 1	Oct. 1
+500	-700	+600

$$> \text{solve}((1+j)*1000.0+(1+10*j/12)*500.0-(1+6*j/12)*700.0+(1+3*j/12)*600.0=1500.0,j);$$

$$0.08219178080 \quad (27)$$

#10. I will receive \$100 one year from now, \$200 two years from now and \$1,000, 10 years from now. What is the present value of this income stream at 4% per year compound interest? What is the present value at 4% per year simple interest? (Simple interest PV is not on this test.)

Compound interest:

$$> i:=.04;$$

$$i:=0.04 \quad (28)$$

$$> (1+i)^{(-1)}*100+(1+i)^{(-2)}*200+(1+i)^{(-10)}*1000;$$

956.6292576

(29)

Simple interest: It is the sum of the present values of each of the payments:

> (1+i)^(-1)*100+(1+2*i)^(-1)*200+(1+10*i)^(-1)*1000;

995.6247457

(30)

11. In the preceding problem, what is the future value 15 years from now of the income stream at 4% per year compound interest? What is the future value 15 years from now at 4% per year simple interest? (Simple interest PV is not on this test.)

Compound interest:

> (1+i)^(14)*100+(1+i)^(13)*200+(1+i)^(5)*1000;

1722.835248

(31)

Simple interest:

> (1+14*i)*100+(1+13*i)*200+(1+5*i)*1000;

1660.00

(32)

#12. How much should you pay for a 15 year Bond with a face value of \$6,000 and quarterly coupons of \$100 if you want a 4% yield compounded quarterly? (Bonds are not on this test.)

> i:=.04/4;

i:= 0.01000000000

(33)

> (1+i)^(-4*15)*(100*((1+i)^(4*15)-1)/i+6000);

7798.201538

(34)

#13. State a loan problem that is equivalent with the bond problem in the preceding problem. How much could you borrow at 4% interest, compounded quarterly, if you are willing to pay \$100 per quarter for 15 years together with a final "balloon" payment of \$6000? (Bonds are not on this test.)