1. Purdue Life sells an annuity that pays $5,000 at the end of the year for 30 years. Assuming that they can invest funds at 8%/year, what is the least they should charge for this annuity—i.e. what is the present value of the payments? Hint: How much would you have at the end of year 30 if you invested $5,000 per year at 8%? The answer will be the present value of this number.

> $i := .08;$

$$i := 0.08 \quad \textbf{(1)}$$

In 30 years you have:

> $A := \frac{5000\cdot\left((1+i)^{30}-1\right)}{i};$

$$A := 5.664160555\ 10^{5} \quad \textbf{(2)}$$

The PV of A is:

> $PV := (1+i)^{-30}\cdot A;$

$$PV := 56288.91671 \quad \textbf{(3)}$$

>

2. I borrow $100,000 which I pay off at a rate of $100/month, paid at the end of the month. At 5% interest compounded monthly, how many months will it take to pay off the loan?

After n months, you owe:

> $i := \frac{.05}{12};$

$$i := 0.004166666667 \quad \textbf{(4)}$$

> $Pn := (1+i)^{n}\cdot 100000;$

$$Pn := 100000\ 1.004166667^{n} \quad \textbf{(5)}$$

You have paid

> $Bn := \frac{100\cdot\left((1+i)^{n}-1\right)}{i};$

$$Bn := 24000.00000\ 1.004166667^{n} - 24000.00000 \quad \textbf{(6)}$$

>

We want to solve $(1+i)^n \cdot 100000 - \frac{100 \cdot ((1+i)^n - 1)}{i} = 0.$

Let $x = (1+i)^n$

> solve$\left(x \cdot 100000 - \frac{100}{i}(x-1) = 0, x\right)$;

$$-0.3157894737 \quad (7)$$

Nonsense! You will never pay off the loan!

I considered this problam as EXTRA CREDIT (5 pts).

3. I borrow $20,000 at 6% interest which I pay off in three annual installments, $6,000 at the end of the first year, $7,000 at the end of the second year, and a final payment at the end of the third year. What should the amount of the final payment be?

You owe:

> $i := .06; P := (1+i)^3 \cdot 20000;$

$$i := 0.06$$
$$P := 23820.32000 \quad (8)$$

Your payments have reduced your debt by:

> $B := 6000 \cdot (1+i)^2 + 7000 \cdot (1+i);$

$$B := 14161.6000 \quad (9)$$

The final payment is:

> $P - B;$

$$9658.72000 \quad (10)$$

Alternate solution:

The present value of all payments must equal the amount of the loan. The PV of the firat two payments is

> $PV := 6000 \cdot (1+i)^{-1} + 7000 \cdot (1+i)^{-2};$

$$PV := 11890.35244 \quad (11)$$

Hence the PV of the last payment is

> PV3 := 20000 − PV;

$$PV3 := 8109.64756 \qquad (12)$$

Hence the last payment is:

> $(1+i)^3 \cdot 8109.64756;$

$$9658.719998 \qquad (13)$$

>

This works because

$(1+i)^3 \cdot (20000-$
$6000 \cdot (1+i)^{-1} + 7000 \cdot (1+i)^{-2}) = (1+i)^3 \cdot 20000 - 6000 \cdot (1+i)^2$
$+ 7000 \cdot (1+i)$

Error, unable to match delimiters

$6000 \cdot (1+i)^{-1} + 7000 \cdot (1+i)^{-2}) = (1+i)^3 \cdot 20000 - 6000 \cdot (1+i)^2 + 7000 \cdot (1+i$

General fact: In a loan

Amount of loan=PV(All payments)