

1) Huntington Bank offers an account that pays 5%, compounded daily. They decide to change to compounding four times a year. What interest rate should they offer to obtain the same annual effective rate as the original account?

$$\begin{aligned} > \left(1 + \frac{i}{4}\right)^4 = \left(1 + \frac{.05}{365}\right)^{365} \\ & \hspace{15em} 1.040604010 = 1.051267381 \end{aligned} \tag{1}$$

$$\begin{aligned} > i := 4 \cdot \left( \left(1 + \frac{.05}{365}\right)^{\frac{365}{4}} - 1 \right) \\ & \hspace{15em} i := 0.050310228 \end{aligned} \tag{2}$$

2) On January 1, I win a prize that pays \$P at the beginning of each month for 10 years with the first payment starting immediately. Find \$P given that the present value of my prize at 4% interest compounded monthly is \$1,000,000.

$$\begin{aligned} > i := \frac{.04}{12}; j := 1 + i; n := 12 \cdot 10 \\ & \hspace{15em} i := 0.003333333333 \\ & \hspace{15em} j := 1.0033333333 \\ & \hspace{15em} n := 120 \end{aligned} \tag{3}$$

$$\begin{aligned} > \text{solve}\left(\frac{(j^n - 1)}{i} \cdot j \cdot j^{-n} \cdot X = 1000000, X\right) \\ & \hspace{15em} 10090.87838 \end{aligned} \tag{4}$$

3) You borrow \$5,000 at the beginning of year 1 at 4% annual effective interest. You pay \$1000 at the end of year 1, and \$2,000 at the end of year 2, \$P at the end of year 3, \$400 at the end of year 4, and \$400 at the end of year 5, after which you owe nothing. Find P.

$$\begin{aligned} > i := .04; \\ & \hspace{15em} i := 0.04 \end{aligned} \tag{5}$$

$$\begin{aligned} > \text{solve}(5000 \cdot (1 + i)^5 = 1000(1 + i)^4 + 2000 \cdot (1 + i)^3 + X \cdot (1 + i)^2 + 400 \cdot (1 + i) + 400, X) \\ & \hspace{15em} 1708.282128 \end{aligned} \tag{6}$$

4) First Bank pays 3% interest, compounded daily. I open an account on January 1 by depositing 10,000. Thereafter, I deposit \$200 at the end of each month for 5 years for a total of 60 deposits. What is the balance in my account immediately after the 60th deposit? Assume that each month has 365/12 days.

$$\begin{aligned} > i := \frac{.03}{365}; j := (1 + i)^{\frac{365}{12}}; ii := j - 1 \\ & \hspace{15em} i := 0.00008219178082 \\ & \hspace{15em} j := 1.002503031 \\ & \hspace{15em} ii := 0.002503031 \end{aligned} \tag{7}$$

$$\begin{aligned} > \text{Ans} := \frac{(j^{60} - 1)}{ii} \cdot 200 + j^{60} \cdot 10000 \\ & \hspace{15em} \text{Ans} := 24548.80028 \end{aligned} \tag{8}$$

$$\begin{aligned} > \text{Ans} := \frac{(j^{60} - 1)}{ii} \cdot 200 + (1 + i)^{5 \cdot 365} \cdot 10000 \\ & \hspace{15em} \text{Ans} := 24548.80049 \end{aligned} \tag{9}$$

5) An account earns 2% annual effective discount for the first two years, 3% annual effective interest for the third year and 4% annual effective force of interest for the last three years. What is the annual

effective interest rate on the account?

$$\begin{aligned} > d := .02; i := .03; del := .04; ii := \frac{d}{1-d}; iii := \exp(del) - 1; \\ & \quad d := 0.02 \\ & \quad i := 0.03 \\ & \quad del := 0.04 \\ & \quad ii := 0.02040816327 \\ & \quad iii := 0.040810774 \end{aligned} \tag{10}$$

$$\begin{aligned} > ans := ((1 + ii)^2 \cdot (1 + i) \cdot (1 + iii)^3)^{\frac{1}{6}} - 1 \\ & \quad ans := 0.032167234 \end{aligned} \tag{11}$$

6) You borrow \$300,000 to buy a house which you finance at 4% annual interest, compounded monthly. How many months will it take to pay off the loan if you pay \$3000 at the end of each month?

$$\begin{aligned} > i := \frac{.04}{12}; j := 1 + i; \\ & \quad i := 0.00333333333333 \\ & \quad j := 1.0033333333 \end{aligned} \tag{12}$$

$$\begin{aligned} > 300000 j^n = \frac{3000 \cdot (j^n - 1)}{i} \\ & \quad 4.472497869 \cdot 10^5 = 4.417493607 \cdot 10^5 \end{aligned} \tag{13}$$

$$\begin{aligned} > \frac{i \cdot 300000}{3000} j^n = j^n - 1 \\ & \quad 0.4969442076 = 0.490832623 \end{aligned} \tag{14}$$

$$\begin{aligned} > q := \frac{i \cdot 300000}{3000} \\ & \quad q := 0.3333333333 \end{aligned} \tag{15}$$

$$\begin{aligned} > jn := \text{solve}((q - 1) \cdot x = -1, x) \\ & \quad jn := 1.500000000 \end{aligned} \tag{16}$$

$$\begin{aligned} > n := \frac{\ln(jn)}{\ln(j)} \\ & \quad n := 121.8421647 \end{aligned} \tag{17}$$

7) You borrow \$150,000 to buy a house which you finance with a 30 year loan at 4% annual interest, compounded monthly on which you pay \$716.12 at the end of each month. How much do you owe at the end of the second year—i.e. immediately after the 24th payment?

$$\begin{aligned} > PP := 716.12 \\ & \quad PP := 716.12 \end{aligned} \tag{18}$$

$$\begin{aligned} > Owe := 150000 \cdot j^{24} - \frac{PP \cdot (j^{24} - 1)}{i} \\ & \quad Owe := 1.446093437 \cdot 10^5 \end{aligned} \tag{19}$$

8) In problem 7, immediately after the 24th payment, I refinance the loan at 3% interest per year. Assuming that the answer to Problem 7 is \$100,000 (which is not correct), find the new annual payment.

$$\begin{aligned}
 > i := \frac{.03}{12}; j := 1 + i; n := 12 \cdot 30 - 24 \\
 & \qquad \qquad \qquad i := 0.002500000000 \\
 & \qquad \qquad \qquad j := 1.002500000 \\
 & \qquad \qquad \qquad n := 336 \qquad \qquad \qquad (20)
 \end{aligned}$$

$$\begin{aligned}
 > Ans := solve\left(0 = 100000 \cdot j^n - \frac{Q \cdot (j^n - 1)}{i}, Q\right) \\
 & \qquad \qquad \qquad Ans := 440.2674240 \qquad \qquad \qquad (21)
 \end{aligned}$$

9) I bought \$50,000 worth of RC Penney stock on January 1. I bought \$5000 worth of RC Penney stock on March 1 and sold \$2000 of RC Penney stock on May 1. On January 1, 2010, I sold all of my RC Penney stock for \$54849.16. Approximate the rate of return on my investment.

This is NOT part of the solution. Here I am creating the problem.:

$$\begin{aligned}
 > i := .035; \\
 & \qquad \qquad \qquad i := 0.035 \qquad \qquad \qquad (22)
 \end{aligned}$$

$$\begin{aligned}
 > 50000 \cdot (1 + i) + 5000 \cdot \left(1 + \frac{10}{12}i\right) - 2000 \cdot \left(1 + \frac{8}{12}i\right) \\
 & \qquad \qquad \qquad 54849.16667 \qquad \qquad \qquad (23)
 \end{aligned}$$

Here is the solution:

$$\begin{aligned}
 > solve\left(50000 \cdot (1 + x) + 5000 \cdot \left(1 + \frac{10}{12}x\right) - 2000 \cdot \left(1 + \frac{8}{12}x\right) = 54849.16, x\right) \\
 & \qquad \qquad \qquad 0.03499986308 \qquad \qquad \qquad (24)
 \end{aligned}$$

10) What price should you pay for a \$4,000 face value, 10 year bond which has \$100 quarterly coupons, assuming that you want a 3% yield, compounded quarterly?

$$\begin{aligned}
 > i := \frac{.03}{4}; j := 1 + i; n := 4 \cdot 10 \\
 & \qquad \qquad \qquad i := 0.007500000000 \\
 & \qquad \qquad \qquad j := 1.007500000 \\
 & \qquad \qquad \qquad n := 40 \qquad \qquad \qquad (25)
 \end{aligned}$$

$$\begin{aligned}
 > P := j^{-n} \cdot \left(\frac{(j^n - 1)}{i} \cdot 100 + 4000\right) \\
 & \qquad \qquad \qquad P := 6411.285687 \qquad \qquad \qquad (26)
 \end{aligned}$$

11) The bond in question (10) is sold after two years, immediately after the payment of the coupon, to an investor wanting a 2% yield. What should the selling price of the bond be?

$$\begin{aligned}
 > i := \frac{.02}{4}; j := 1 + i; n := 4 \cdot 8 \\
 & \qquad \qquad \qquad i := 0.005000000000 \\
 & \qquad \qquad \qquad j := 1.005000000 \\
 & \qquad \qquad \qquad n := 32 \qquad \qquad \qquad (27)
 \end{aligned}$$

$$\begin{aligned}
 > P := j^{-n} \cdot \left(\frac{(j^n - 1)}{i} \cdot 100 + 4000\right) \\
 & \qquad \qquad \qquad P := 6360.262689 \qquad \qquad \qquad (28)
 \end{aligned}$$

12) At the beginning of year 1 I deposit \$1000 into an account that is earning 4% interest compounded

annually. At the beginning of each subsequent year I deposit 3 % more than I did the previous year.  
Find the final accumulation in the account at the end of year 40.

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> i := .04; j := 1 + i; ii := .03; jj := 1 + ii; P := 1000
      i := 0.04
      j := 1.04
      ii := 0.03
      jj := 1.03
      P := 1000
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(29)

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> A = P·j40 + jj·P·j39 + jj2·P·j38 + ... + jj39·P·j
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Error, invalid sum/difference

$$A = P \cdot j^{40} + jj \cdot P \cdot j^{39} + jj^2 \cdot P \cdot j^{38} + \dots + jj^{39} \cdot P \cdot j$$

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> = jj39·j·P·( (j/jj)39 + (j/jj)38 + ... + 1 )
```

Error, invalid sum/difference

$$= jj^{39} \cdot j \cdot P \cdot \left( \left( \frac{j}{jj} \right)^{39} + \left( \frac{j}{jj} \right)^{38} + \dots + 1 \right)$$

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> Ans := 
$$\frac{jj^{39} \cdot j \cdot P \cdot \left( \left( \frac{j}{jj} \right)^{40} - 1 \right)}{\frac{j}{jj} - 1}$$

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Ans := 1.600542126 10<sup>5</sup>

(30)

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