1) Huntington Bank offers an account that pays 4%, compounded daily. They decide to change to compounding four times a year. What interest rate should they offer to obtain the same annual effective rate as the original account?

\[
\left( 1 + \frac{i}{4} \right)^4 = \left( 1 + \frac{0.04}{365} \right)^{365} \\
\left( 1 + \frac{i}{4} \right)^4 = 1.040808478
\]

\[
i := 4 \cdot \left( \left( 1 + \frac{0.04}{365} \right)^{\frac{365}{4}} - 1 \right) \\
i := 0.040198440
\]

2) On January 1, I win a prize that pays $P$ at the beginning of each month for 10 years with the first payment starting immediately. Find $P$ given that the present value of my prize at 3% interest compounded monthly is $1,000,000.

\[
i := \frac{0.03}{12}; j := 1 + i; n := 12 \cdot 10 \\
i := 0.002500000000 \\
j := 1.002500000 \\
n := 120
\]

\[
solve \left( \frac{j^n - 1}{i} \cdot j^{-n} \cdot x = 1000000, x \right) \\
x = 9631.994490
\]

3) You borrow $5,000 at the beginning of year 1 at 3% annual effective interest. You pay $1000 at the end of year 1, and $2,000 at the end of year 2, $P$ at the end of year 3, $400$ at the end of year 4, and $400$ at the end of year 5, after which you owe nothing. Find $P$.

\[
i := .03; \\
i := 0.03
\]

\[
solve \left( 5000 \cdot (1 + i)^5 = 1000(1 + i)^4 + 2000 \cdot (1 + i)^3 + P \cdot (1 + i)^2 + 400 \cdot (1 + i) + 400, P \right) \\
P = 1577.347120
\]

4) First Bank pays 4% interest, compounded daily. I open an account on January 1 by depositing 10,000. Thereafter, I deposit $200 at the end of each month for 5 years for a total of 60 deposits. What is the balance in my account immediately after the 60th deposit? Assume that each month has 365/12 days.

\[
i := \frac{0.04}{365}; j := (1 + i)^{\frac{365}{12}}; ii := j - 1 \\
i := 0.0001095890411 \\
j := 1.003338711 \\
ii := 0.003338711
\]

\[
\text{Ans} := \left( \frac{j^{60} - 1}{ii} \right) \cdot 200 + j^{60} \cdot 10000 \\
\text{Ans} := 25475.85645
\]

\[
\text{Ans} := \left( \frac{j^{60} - 1}{ii} \right) \cdot 200 + (1 + i)^{5 \cdot 365} \cdot 10000 \\
\text{Ans} := 25475.85613
\]
5) An account earns 5% annual effective discount for the first two years, 3% annual effective interest for the third year and 4% annual effective force of interest for the last three years. What is the annual effective interest rate on the account?

\[ d := .05; \quad i := .03; \quad del := .04; \quad ii := \frac{d}{1 - d}; \quad iii := \exp(del) - 1; \]

\[ d := 0.05 \]
\[ i := 0.03 \]
\[ del := 0.04 \]
\[ ii := 0.05263157895 \]
\[ iii := 0.040810774 \]

\[ ans := \left( (1 + ii)^2 \cdot (1 + i) \cdot (1 + iii)^3 \right)^{\frac{1}{6}} - 1 \]

\[ ans := 0.042919750 \]  

6) You borrow $300,000 to buy a house which you finance at 3% annual interest, compounded monthly. How many months will it take to pay off the loan if you pay $3000 at the end of each month?

\[ i := \frac{.03}{12}; \quad j := 1 + i; \]
\[ i := 0.002500000000 \]
\[ j := 1.002500000 \]

\[ 300000 \cdot j^n = \frac{3000 \cdot (j^n - 1)}{i} \]

\[ 4.048060641 \cdot 10^5 = 4.192242564 \cdot 10^5 \]

\[ \frac{i \cdot 300000}{3000} \cdot j^n = j^n - 1 \]

\[ 0.3373383868 = 0.349353547 \]

\[ q := \frac{i \cdot 300000}{3000} \]

\[ q := 0.2500000000 \]

\[ jn := \text{solve}\left((q - 1) \cdot x = -1, x\right) \]
\[ jn := 1.333333333 \]

\[ n := \frac{\ln(jn)}{\ln(j)} \]

\[ n := 115.2166100 \]

7) You borrow $150,000 to buy a house which you finance with a 30 year loan at 4% annual interest, compounded monthly on which you pay $632.40 at the end of each month. How much do you owe at the end of the second year—i.e. immediately after the 24th payment?

\[ PP := 632.40 \]

\[ PP := 632.40 \]

\[ Owe := 150000 \cdot j^{24} - \frac{PP \cdot (j^{24} - 1)}{i} \]

\[ Owe := 1.466975620 \cdot 10^5 \]

8) In problem 7, immediately after the 24th payment, I refinance the loan at 2% interest per year. Assuming that the answer to Problem 7 is $100,000 (which is not correct), find the new annual payment.
\[
\begin{align*}
\text{\texttt{\textgreater i := \frac{.02}{12}; j := 1 + i; n := 12 \cdot 30 - 24}} \\
&\quad i := 0.001666666667 \\
&\quad j := 1.001666667 \\
&\quad n := 336 \\
\text{\texttt{\textgreater Ans := solve\left(0 = 100000 \cdot j^n - \frac{Q \cdot (j^n - 1)}{i}, Q\right)}} \\
&\quad Ans := 388.9313232 \quad (21)
\end{align*}
\]

9) I bought $50,000 bought of RC Penney stock on January 1. I bought $5000 worth of RC Penney stock on March 1 and sold $2000 of RC Penney stock on May 1. At the end of the year I sold all of my RC Penney stock for $54,320.83. Approximate the rate of return on my investment.

This is NOT part of the solution. Here I am creating the problem:

\[
\begin{align*}
\text{\texttt{\textgreater i := .025;}} \\
&\quad i := 0.025 \\
\text{\texttt{\textgreater 50000 \cdot (1 + i) + 5000 \cdot \left(1 + \frac{10}{12} \cdot i\right) - 2000 \cdot \left(1 + \frac{8}{12} \cdot i\right)}} \\
&\quad 54320.83333 \\
\end{align*}
\]

Here is the solution:

\[
\begin{align*}
\text{\texttt{\textgreater solve\left(50000 \cdot (1 + x) + 5000 \cdot \left(1 + \frac{10}{12} \cdot x\right) - 2000 \cdot \left(1 + \frac{8}{12} \cdot x\right) = 54320.83, x\right)}} \\
&\quad 0.02499993691 \quad (24)
\end{align*}
\]

10) What price should you pay for a $4,000 face value, 10 year bond which has $100 quarterly coupons, assuming that you want a 2% yield, compounded quarterly?

\[
\begin{align*}
\text{\texttt{\textgreater i := \frac{.02}{4}; j := 1 + i; n := 4 \cdot 10}} \\
&\quad i := 0.005000000000 \\
&\quad j := 1.005000000 \\
&\quad n := 40 \\
\text{\texttt{\textgreater P := j^{-n} \cdot \left(\frac{j^n - 1}{i}\right) \cdot 100 + 4000}}} \\
&\quad P := 6893.778221 \quad (26)
\end{align*}
\]

11) The bond in question (10) is sold after two years, immediately after the payment of the coupon, to an investor wanting a 1% yield. What should the selling price of the bond be?

\[
\begin{align*}
\text{\texttt{\textgreater i := \frac{.01}{4}; j := 1 + i; n := 4 \cdot 8}} \\
&\quad i := 0.002500000000 \\
&\quad j := 1.002500000 \\
&\quad n := 32 \\
\text{\texttt{\textgreater P := j^{-n} \cdot \left(\frac{j^n - 1}{i}\right) \cdot 100 + 4000}}} \\
&\quad P := 6764.493690 \quad (28)
\end{align*}
\]

12) At the beginning of year 1 I deposit $1000 into an account that is earning 5% interest compounded
annually. At the beginning of each subsequent year I deposit 3% more than I did the previous year. Find the final accumulation in the account at the end of year 40.

\[ i := .05; j := 1 + i; ii := .03; jj := 1 + ii; P := 1000 \]

\[ i := 0.05 \]
\[ j := 1.05 \]
\[ ii := 0.03 \]
\[ jj := 1.03 \]
\[ P := 1000 \]

\[ A = P \cdot j^{40} + jj \cdot P \cdot j^{39} + jj^2 \cdot P \cdot j^{38} + \ldots + jj^{39} \cdot P \cdot j \]

Error, invalid sum/difference

\[ A = P \cdot j^{40} + jj \cdot P \cdot j^{39} + jj^2 \cdot P \cdot j^{38} + \ldots + jj^{39} \cdot P \cdot j \]

\[ = jj^{39} \cdot P \cdot \left( \left( \frac{j}{jj} \right)^{39} + \left( \frac{j}{jj} \right)^{38} + \ldots + 1 \right) \]

Error, invalid sum/difference

\[ = jj^{39} \cdot P \cdot \left( \left( \frac{j}{jj} \right)^{39} + \left( \frac{j}{jj} \right)^{38} + \ldots + 1 \right) \]

\[ Ans := \frac{jj^{39} \cdot j \cdot P}{jj} \left( \left( \frac{j}{jj} \right)^{40} - 1 \right) \]

\[ \frac{j}{jj} - 1 \]

\[ Ans := 1.983424206 \times 10^5 \]