1) You receive an award that pays $1,000 at the beginning of year 0, $P$ at the beginning of year 1, and $3,000 at the beginning of year 2. Find $P$, given that at the beginning of year 0, the present value of the award at 5% interest per year was $9300.

Total present values of the award and equate to 9300

\[ i := .05; \ PV := 1000 + (1 + i)^{(1)} \cdot P + (1 + i)^{(2)} \cdot 3000 \]

\[ i := 0.05 \]

\[ PV := 3721.088436 + 0.9523809524 \cdot P \]  

(1)

Equate with 9300 and solve for $P$

\[ solve(9300 = PV, P) \]

\[ 5857.857142 \]  

(2)

2) I invest $P$ at the end of the year for 30 years at 4% interest compounded annually. Find $P$, given that the total the present value of all of the deposits is $7781.41.

\[ i := .04; n := 30 \]

\[ i := 0.04 \]

\[ n := 30 \]  

(3)

\[ FV := \left(\frac{(1 + i)^n - 1}{i}\right) \cdot P \]

\[ FV := 56.08493775 \cdot P \]  

(4)

\[ PV := (1 + i)^{-n} \cdot FV \]

\[ PV := 17.29203330 \]  

(5)

\[ solve(PV = 7781.41, P) \]

\[ 449.9997117 \]  

(6)

3) Huntington Bank offers an account that pays 5%, compounded monthly. They decide to change to daily compounding. What interest rate should they offer to obtain the same annual effective rate as the original account? State your answer as a nominal annual rate--i.e. $i\%$ per year, compounded daily.

\[ \left(1 + \frac{.05}{12}\right)^{12} = \left(1 + \frac{i}{365}\right)^{365} \]

\[ i := 365 \cdot \left(\left(1 + \frac{.05}{12}\right)^{12} - 1\right); Ans := 100 \cdot i \]

\[ i := 0.049899515 \]

\[ Ans := 4.989951500 \]  

(7)

4) What is the least amount I can have in my retirement fund to allow me to withdraw $5,000 at the end of each month for the next 40 years, assuming that my fund earns 4% interest, compounded monthly?

Solution: The future value of my withdrawals (FVWD) must equal the future value (FVB) of $5000

\[ i := \frac{.04}{12}; n := 40 \cdot 12 \]

\[ i := 0.003333333333 \]

\[ n := 480 \]  

(8)
5) You borrow $50,000 to buy a car which you finance at 6% annual interest, compounded monthly. How many months will it take to pay off the loan if you pay $500 at the end of each month?

\[
i := \frac{.06}{12}; j := 1 + i
\]

\[
FV(50000) = FV(500)
\]

\[
50000 \cdot j^n = \frac{(j^n - 1)}{i} \cdot 500, j := 1 + i
\]

\[
i \cdot 100 \cdot j^n = j^n - 1
\]

\[
1 = j^n (1 - 100 \cdot i) = .5 \cdot j^n
\]

\[
j^n = \frac{1}{.5} = .2
\]

\[
n := \frac{\ln(2.0)}{\ln(j)} = 138.9757216
\]

\[
> i := \frac{.06}{12}; j := 1 + i
\]

\[
i := 0.005000000000
\]

\[
j := 1.005000000
\]

\[
> n := \frac{\ln(2.0)}{\ln(j)}
\]

\[
n := 138.9757216
\]

We round up: Ans=139 months.

6) I borrow $200,000 for 30 years at 7% interest per year with annual payments of $16117.28 made at the end of the year. What do I still owe immediately after the 10th payment?

\[
FV(200000) - FV(payments)
\]

\[
i := .07; j := 1 + i; n := 10; p := 16117.28; Ans := 200000 \cdot j^n - \frac{(j^n - 1)}{i} \cdot p
\]

\[
i := 0.07
\]

\[
j := 1.07
\]

\[
n := 10
\]

\[
p := 16117.28
\]

\[
Ans := 1.707467110 \times 10^5
\]
7) In problem 5, immediately after the 10th payment, I refinance the loan at 4% interest per year. Assuming that the answer to Problem 6 is $150,000 (which is not correct), find the new annual payment.

\[ \text{FV}(150000) - \text{FV} (\text{payments}) \]

\[ i := 0.04; \quad j := 1 + i; \quad n := 30 - 10; \]

\[ i := 0.04 \]
\[ j := 1.04 \]
\[ n := 20 \]  

\[ \text{Ans} := \text{solve} \left( 150000 \cdot j^n = \frac{(j^n - 1)}{i} \cdot P, P \right) \]

\[ \text{Ans} := 11037.26255 \]  

8) I bought $1,000 bought of Purdue stock on January 1. I sold $200 worth of Purdue stock on June 1 and bought $500 of Purdue stock on July 1. At the end of the year I sold all of my Purdue stock for $1400. Approximate the rate of return on my investment.

My $200 lost 7 months of interest: June-Dec.

My $500 earned 6 months of interest: July-Dec.

\[ \text{Ans} := \text{solve} \left( 1000.0 \cdot (1 + \text{int}) - 200.0 \cdot \left(1 + 7 \cdot \frac{\text{int}}{12}\right) + 500.0 \cdot \left(1 + 6 \cdot \frac{\text{int}}{12}\right) = 1400.0, \text{int} \right) \]

\[ \text{Ans} := 0.08823529414 \]  

9) What price should you pay for a $5,000 face value, 20 year bond which has $100 quarterly coupons, assuming that you want a 4% yield, compounded quarterly? (Thus you receive 20*4=80 payments of $100 at the end of each quarter for 20 years, plus a final payment of $5,000.)

\[ \text{Price} = \text{PV}(\text{All payments}) \]

\[ i := \frac{0.04}{4}; \quad j := 1 + i; \quad n := 4 \cdot 20; \quad \text{Ans} := j^{-n} \cdot 5000 + j^{-n} \cdot \frac{(j^n - 1)}{i} \cdot 100 \]

\[ i := 0.01000000000 \]
\[ j := 1.010000000 \]
\[ n := 80 \]

\[ \text{Ans} := 7744.410303 \]  

10) I sale the bond in problem 9 immediately after the payment of the 20th payment to a buyer wanting a 5% yield. What price do I charge?

\[ \text{Price} = \text{PV}(\text{All payments}) \]

\[ i := \frac{0.05}{4}; \quad j := 1 + i; \quad n := 4 \cdot 20 - 20; \quad \text{Ans} := j^{-n} \cdot 5000 + j^{-n} \cdot \frac{(j^n - 1)}{i} \cdot 100 \]

\[ i := 0.01250000000 \]
\[ j := 1.012500000 \]
\[ n := 60 \]

\[ \text{Ans} := 6576.297193 \]