1) You receive an award that pays $1,000 at the beginning of year 0, $P at the beginning of year 1, and $3,000 at the beginning of year 2. Find P, given that at the beginning of year 0, the present value of the award at 5% interest per year was $9300.

Total present values of the award and equate to 9300

> $i := .05; PV := 1000 + (1+i)\text{^}(-1) * P + (1+i)\text{^}(-2) * 3000$

$$i := 0.05$$

$$PV := 3721.088436 + 0.9523809524\,P \quad \textbf{(1)}$$

Equate with 9300 and solve for P

> $solve(9300 = PV, P)$

$$5857.857142 \quad \textbf{(2)}$$

2)I invest $P at the end of the year for 30 years at 4% interest compounded annually. Find P, given that the total the present value of all of the deposits is $7781.41.

> $i := .04; n := 30$

$$i := 0.04$$

$$n := 30 \quad \textbf{(3)}$$

> $FV := \frac{\left((1+i)^n - 1\right)}{i} P$

$$FV := 56.08493775\,P \quad \textbf{(4)}$$

> $PV := (1+i)^{-n} \cdot FV$

$$PV := 17.29203330\,P \quad \textbf{(5)}$$

> $solve(PV = 7781.41, P)$

$$449.9997117 \quad \textbf{(6)}$$

3)Huntington Bank offers an account that pays 5%, compounded monthly. They decide to change to daily compounding. What interest rate should they offer to obtain the same annual effective rate as the original account? State your answer as a nominal annual rate--i.e. i% per year, compounded daily.

$$\left(1 + \frac{.05}{12}\right)^{12} = \left(1 + \frac{i}{365}\right)^{365}$$

> $i := 365 \cdot \left(\left(1 + \frac{.05}{12}\right)^{\frac{12}{365}} - 1\right); Ans := 100 \cdot i$

$$i := 0.049899515$$

$$Ans := 4.989951500 \quad \textbf{(7)}$$

4)What is the least amount I can have in my retirement fund to allow me to withdraw $5,000 at the end of each month for the next 40 years, assuming that my fund earns 4% interest, compounded monthly?

Solution: The future value of my withdrawals (FVWD) must equal the future value (FVB) of $5000

> $i := \frac{.04}{12}; n := 40 \cdot 12$

$$i := 0.003333333333$$

$$n := 480 \quad \textbf{(8)}$$

> $FVWD := \frac{((1+i)^n - 1)}{i} \cdot 5000$

$$FVWD := 5.909805520\ 10^6 \quad \textbf{(9)}$$

> $FVB := (1+i)^n \cdot Bal$

$$FVB := 4.939870345\ Bal \quad \textbf{(10)}$$

> $Ans := solve(FVWD = FVB, Bal)$

$$Ans := 1.196348306\ 10^6 \quad \textbf{(11)}$$

5)You borrow $50,000 to buy a car which you finance at 6% annual interest, compounded monthly. How many months will it take to pay off the loan if you pay $500 at the end of each month?

$i := \frac{.06}{12}; \mathrm{j} := 1 + \mathrm{i}$

FV(50000)=FV(500)

$50000 \cdot j^n = \frac{(j^n - 1)}{i} \cdot 500, \mathrm{j} := 1 + \mathrm{i}$

$i \cdot 100 \cdot j^n = j^n - 1$

$1 = \mathrm{j}^n(1 - 100 \cdot i) = .5 \cdot j^n$

$j^n = \frac{1}{.5} = .2$

$n := \frac{\ln(2.0)}{\ln(j)} = 138.9757216$

> $i := \frac{.06}{12}; \mathrm{j} := 1 + \mathrm{i}$

$$i := 0.005000000000$$
$$j := 1.005000000 \quad \textbf{(12)}$$

> $n := \frac{\ln(2.0)}{\ln(j)}$

$$n := 138.9757216 \quad \textbf{(13)}$$

We round up: Ans=139 months.

6)I borrow $200,000 for 30 years at 7% interest per year with annual payments of $16117.28 made at the end of the year. What do I still owe immediately after the 10th payment?
FV(200000)-FV(payments)

> $i := .07; j := 1 + i; n := 10; p := 16117.28; Ans := 200000 \cdot j^n - \frac{(j^n - 1)}{i} \cdot p$

$$i := 0.07$$
$$j := 1.07$$
$$n := 10$$
$$p := 16117.28$$
$$Ans := 1.707467110\ 10^5 \quad \textbf{(14)}$$

7)In problem 5, immediately after the 10th payment, I refinance the loan at 4% interest per year. Assuming that the answer to Problem 6 is $150,000 (which is not correct), find the new annual payment.

FV(150000)-FV(payments)

> $i := .04; j := 1 + i; n := 30 - 10;$

$$i := 0.04$$
$$j := 1.04$$
$$n := 20 \quad \textbf{(15)}$$

> $Ans := solve\left(150000 \cdot j^{n} = \frac{\left(j^{n} - 1\right)}{i} \cdot P, P\right)$

$$Ans := 11037.26255 \quad \textbf{(16)}$$

8)I bought $1,000 bought of Purdue stock on January 1. I sold $200 worth of Purdue stock on June 1 and bought $500 of Purdue stock on July 1. At the end of the year I sold all of my Purdue stock for $1400. Approximate the rate of return on my investment.

My $200 lost 7 months of interest: June-Dec.

My $500 earned 6 months of interest: July-Dec.

> $Ans := solve\left(1000.0 \cdot (1 + int) - 200.0 \cdot \left(1 + 7 \cdot \frac{int}{12}\right) + 500.0 \cdot \left(1 + \frac{6 \cdot int}{12}\right) = 1400.0, int\right)$

$$Ans := 0.08823529414 \quad \textbf{(17)}$$

9) What price should you pay for a $5,000 face value, 20 year bond which has $100 quarterly coupons, assuming that you want a 4% yield, compounded quarterly? (Thus you receive 20*4=80 payments of $100 at the end of each quarter for 20 years, plus a final payment of $5,000.)

Price=PV(All payments)

> $i := \frac{.04}{4}; j := 1 + i; n := 4 \cdot 20; Ans := j^{-n} \cdot 5000 + j^{-n} \cdot \frac{\left(j^{n} - 1\right)}{i} \cdot 100$

$$i := 0.01000000000$$
$$j := 1.010000000$$
$$n := 80$$
$$Ans := 7744.410303 \quad \textbf{(18)}$$

10) I sale the bond in problem 9 immediately after the payment of the 20th payment to a buyer wanting a 5% yield. What price do I charge?

Price=PV(All payments)

> $i := \frac{.05}{4}; j := 1 + i; n := 4 \cdot 20 - 20; Ans := j^{-n} \cdot 5000 + j^{-n} \cdot \frac{\left(j^{n} - 1\right)}{i} \cdot 100$

$$i := 0.01250000000$$
$$j := 1.012500000$$
$$n := 60$$
$$Ans := 6576.297193 \quad \textbf{(19)}$$