CHAPTER 1

Some Useful Formulas

1. Variable Payments

Assume that we receive a series of payments at the end of the year where each payment increases (or decreases) by a factor of k each year. Thus if the first payment is P, then the subsequent payments would be $kP, k^2P, k^3P, \ldots$.

THEOREM 1. *Under the payment scheme described above, if we deposit the payments into an account earning interest i, then the total accumulation after the n^{th} payment is*

$$\begin{aligned} A &= Pk^{n-1}\frac{\left(\frac{j}{k}\right)^n - 1}{\frac{j}{k} - 1} \\ &= Pj^{n-1}\frac{\left(\frac{k}{j}\right)^n - 1}{\frac{k}{j} - 1} \\ &= P\frac{j^n - k^n}{j - k} \quad \textit{where} \\ j &= 1 + i \end{aligned}$$

Proof Our payments accumulate to

$$\begin{aligned} A &= Pj^{n-1} + Pkj^{n-2} + \cdots + Pj^0k^{n-1} \\ &= Pk^{n-1}\left(\left(\frac{j}{k}\right)^{n-1} + \left(\frac{j}{k}\right)^{n-2} + \cdots + 1\right) \\ &= Pk^{n-1}\frac{\left(\frac{j}{k}\right)^n - 1}{\frac{j}{k} - 1} \\ &= Pk^{n-1}\frac{k\left(\left(\frac{j}{k}\right)^n - 1\right)}{j - k} \\ &= P\frac{j^n - k^n}{j - k} \end{aligned}$$

proving the first and third equality in the theorem. (We used formula (7) on page 8 of the notes with $x = \frac{j}{k}$ in step 3.)

The second equality follows by factoring Pj^{n-1} out of the sum instead of Pk^{n-1}. □

Our solution to Example 1 on the 2009 Final is a good example of the use of this formula.

2. Disappearing Deductibles

On p. 29 Brown gives an example of a disappearing deductible. He does not, however, give a general formula. Here it is:

Assume that up to A dollars, we have a deductible of D_o. Our deductible disappears linearly for claims between A and B, vanishing at B. Then for a claim of X between A and B the deductible is

$$D = \frac{B - X}{B - A} D_o.$$

You can check that this is correct by noting that at $X = A$ this formula yields D_o while at $X = B$, it yields 0.